$\boxed{1}$ (20 points)

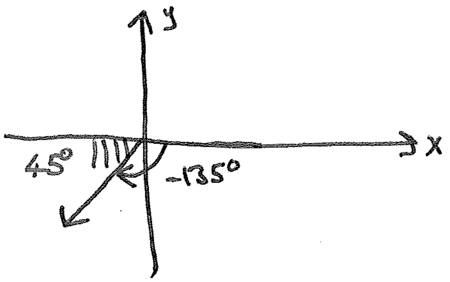
(a) (10 points) Find the value of $2 + \sin^2(75°) + \sin^2(15°)$. Explain your answer.

$= 2 + \sin^2(75°) + \cos^2(75)$ cofunctions

$= 2 + 1$ by Pythagorean Identity

$= 3$

(b) (10 points) Use the reference angle to find the exact value of $\sin(-135°)$. Explain your answer.



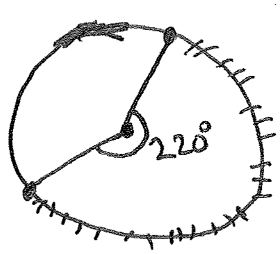
The refrence angle of $-135°$ is $45°$.

So, $\sin(-135°) = \mp \sin(45°)$.

But, we choose the negative sign because sin is negative in the third quadrant.

$\sin(-135°) = - \sin(45°) = -\frac{\sqrt{2}}{2}$

$\boxed{2}$ (10 points) A circle has a radius 9 feet. Find the length of the arc intercepted by a central angle of 220°.



The formula is $s = r \cdot \theta$ where r is the radius and θ is radian measure of central angle of the circle.

$\theta = 220° = 220° \cdot \frac{\pi}{180°} = \frac{11\pi}{9}$ radian.

$s = \cancel{9} \cdot \frac{11\pi}{\cancel{9}} = 11\pi$.

$\boxed{3}$ (15 points) Determine the amplitude, period, and phase shift of $y = \frac{1}{2}\sin(x - \frac{\pi}{4})$. Then graph the function (you should graph the function for more than one period).

$A = \frac{1}{2}$, $B = 1$, $c = \frac{\pi}{4}$.

$$\text{Amplitude} = \left|\tfrac{1}{2}\right| = \tfrac{1}{2}$$

$$\text{period} = \frac{2\pi}{B} = 2\pi$$

$$\text{phase shift} = \frac{c}{B} = \frac{\frac{\pi}{4}}{1} = \frac{\pi}{4}$$

So, the key points are

quarter of the period

$x_1 = \frac{\pi}{4}$

$x_2 = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$

$x_3 = \frac{3\pi}{4} + \frac{\pi}{2} = \frac{5\pi}{4}$

$x_4 = \frac{5\pi}{4} + \frac{\pi}{2} = \frac{7\pi}{4}$

$x_5 = \frac{7\pi}{4} + \frac{\pi}{2} = \frac{9\pi}{4}$.

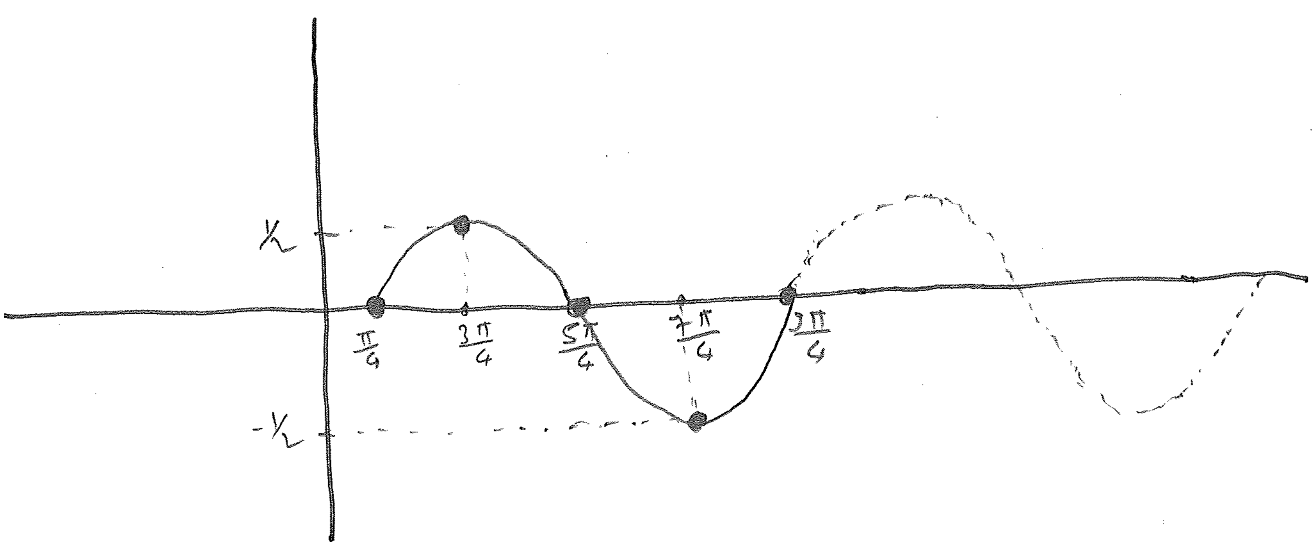
$y_1 = \frac{1}{2}(\sin \frac{\pi}{4} - \frac{\pi}{4}) = 0$

$y_2 = \frac{1}{2}(\sin \frac{3\pi}{4} - \frac{\pi}{4}) = \frac{1}{2}$

$y_3 = \frac{1}{2} \sin \frac{5\pi}{4} - \frac{\pi}{4} = 0$

$y_4 = \frac{1}{2} \sin \frac{7\pi}{4} - \frac{\pi}{4} = -\frac{1}{2}$

$y_5 = \frac{1}{2} \sin \frac{9\pi}{4} - \frac{\pi}{4} = 0$



$\boxed{4}$ (15 points) Graph the function $y = 2\tan(x - \frac{\pi}{2})$.

- First, we need to find the vertical asymptotes

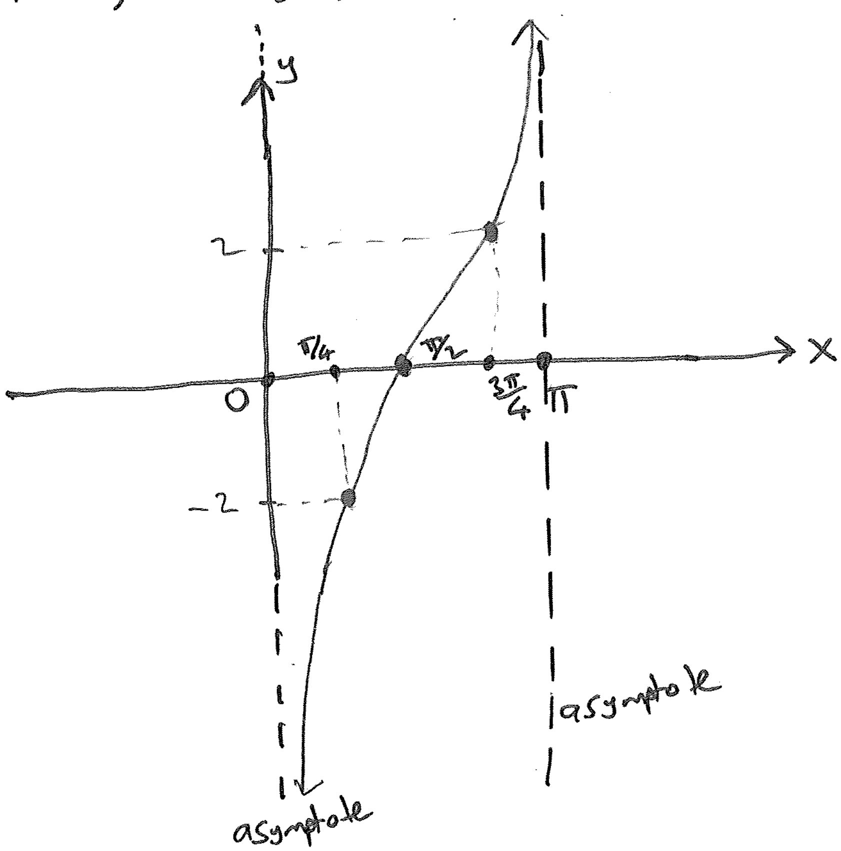
$$-\frac{\pi}{2} < x - \frac{\pi}{2} < \frac{\pi}{2}$$

$$0 < x < \pi$$

so, one asymptote occurs at $x=0$ and the other one occurs at $x=\pi$.

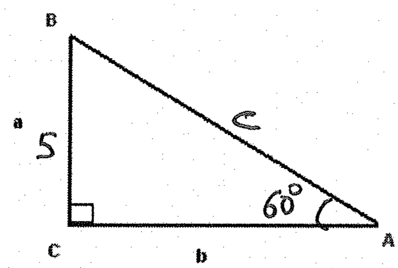
- The x-intercept occurs in the midway of two consecutive asymptotes, so at $x=\frac{\pi}{2}$.

- At $\frac{1}{4}$ and $\frac{3}{4}$ of the way between consecutive asymptotes, the graph takes values $A=2$ and $-A=-2$.



5 (15 points) Use the right triangle shown in the picture to find b, c, and B. We know that $a = 5$, $A = 60°$. You need to use trigonometric functions for this question, other methods will be disregarded.

One way to do it :



- $\sin 60° = \frac{a}{c}$

$\frac{\sqrt{3}}{2} = \frac{5}{c}$

$c = \frac{5}{\frac{\sqrt{3}}{2}} = \frac{10}{\sqrt{3}} = \boxed{\frac{10\sqrt{3}}{3}}$.

- $\cos 60° = \frac{b}{c}$

$\frac{1}{2} = \frac{b}{\frac{10\sqrt{3}}{3}}$

$b = \boxed{\frac{10\sqrt{3}}{6}}$

$A + B = 90°$

$60° + B = 90°$

$\boxed{B = 30°}$

6 (25 points)

(a) (4 points) Find the exact value of $\cos^{-1}(-\frac{\sqrt{2}}{2})$. Explain your answer.

let $\cos^{-1}(-\frac{\sqrt{2}}{2}) = \theta$. So, we are looking for an angle in $[0, \pi]$ such that $\cos\theta = -\frac{\sqrt{2}}{2}$.
θ has to be in quadrant II.
So, θ is $135° = \frac{3\pi}{4}$. Thus, $\cos^{-1}(-\frac{\sqrt{2}}{2}) = \frac{3\pi}{4}$.

(b) (3 points) Find the exact value of $\cos(\cos^{-1}(0.4))$. Explain your answer.

Since 0.4 in the interval $[-1, 1]$, then
$\cos(\cos^{-1}(0.4)) = 0.4$.

(c) (3 points) Find the exact value of $\sin^{-1}(-2)$. Explain your answer.

There is no angle θ such that $\sin\theta = -2$ becouse the ronge of sin is $[-1, 1]$.

So, $\sin^{-1}(-2) =$ undefined.

(d) (3 points) Find the exact value of $\sin^{-1}\left(\sin(\frac{2\pi}{3})\right)$. Explain your answer.

$\frac{2\pi}{3}$ is not in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

$\sin(\frac{2\pi}{3}) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ by the reference angle idea.

$\sin^{-1}(\sin\frac{2\pi}{3}) = \sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$ becouse $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

(e) (3 points) Find the exact value of $\tan\left(\tan^{-1}(12)\right)$. Explain your answer.

$\tan(\tan^{-1}x) = x$ for every real number x.

So, $\tan(\tan^{-1}(12)) = 12$.

(f) (4 points) Find the exact value of $\tan^{-1}\left(\tan(\frac{4\pi}{3})\right)$. Explain your answer.

$\frac{4\pi}{3}$ is <u>not</u> in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$.

$\tan(\frac{4\pi}{3}) = \tan\frac{\pi}{3}$ by the reference angle idea.

$\Rightarrow \tan^{-1}(\tan\frac{\pi}{3}) = \frac{\pi}{3}$ becouse $\frac{\pi}{3} \in (-\frac{\pi}{2}, \frac{\pi}{2})$

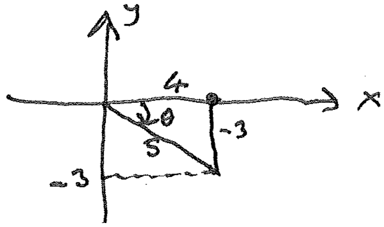
(g) (5 points) Find the exact value of $\tan\left(\sin^{-1}(-\frac{3}{5})\right)$. Explain your answer.

Let $\theta = \sin^{-1}(-\frac{3}{5})$, so we are looking for $\tan\theta$.

$\Downarrow$

$\sin\theta = -\frac{3}{5}$ and $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

but since sin is negotive θ has to be $[-\frac{\pi}{2}, 0]$.



Therefore, by Pythogeran theorem $x = 4$.

And, $\tan\theta = -\frac{3}{4}$.