MAC 1114

EXAM ONE

FALL 2016

Name

Signature

Solutions.

Problem	Total Points	Score
1	15	
2	10	
3	15	
4	10	
5	30	
6	15	
7	10	
Total	105	

- You are not permitted to use a calculator on this exam.
- In order to receive full credit, you must show your work. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Good luck!

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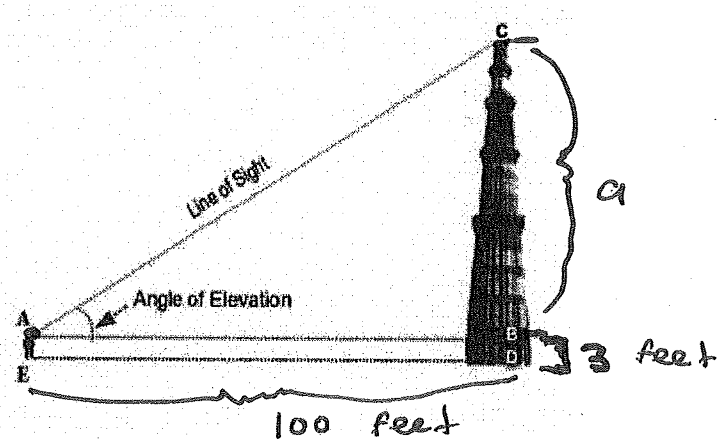
1 (15 points)

(a) (5 points) Circle the correct one:
Reference angle of an angle is always an **acute** / **obtuse** / **straight** /**quadrantal** angle.

(b) (10 points) A surveyor measured the angle of elevation to be $60°$. The transit is 3 feet above the ground (that is the distance between B and D in the graph) and 100 feet from the tower. Find the height of the tower.



$\tan 60° = \frac{a}{100}$

$a = 100 \cdot \tan 60° = 100 \cdot \sqrt{3}$

So, the height is

$\boxed{100 \cdot \sqrt{3} + 3}$

2 (10 points)

(a) (5 points) Find the value of $\cos^2(15°) + \cos^2(75°) - 3$.

By cofunction idea, $\cos^2(15°) = \sin^2(75°)$

So, it is $\underbrace{\sin^2(75°) + \cos^2(75°)}_{=1 \text{ by Pythagorean}} - 3 = 1 - 3 = \boxed{-2}$

(b) (5 points) Find the value of $\cos(30°) + \sin(\frac{\pi}{6}) + \tan(\frac{\pi}{4})$.

$\frac{\sqrt{3}}{2} + \frac{1}{2} + 1 = \frac{3+\sqrt{3}}{2}$

$\tan \frac{\pi}{4} = \frac{\sin \pi/4}{\cos \pi/4} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

3 (15 points)

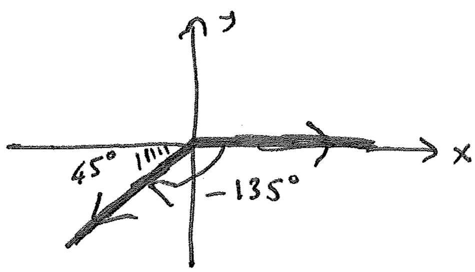
(a) (5 points) Convert $-\frac{3\pi}{4}$ to degrees.

$$-\frac{3\pi}{4} \cdot \frac{180^\circ}{\cancel{\pi}} = -135^\circ$$

(b) (5 points) Find a positive angle less than 360° that is coterminal with −135°.

$$-135^\circ + 360^\circ = 225^\circ$$

(c) (5 points) Find the reference angle of −135°.



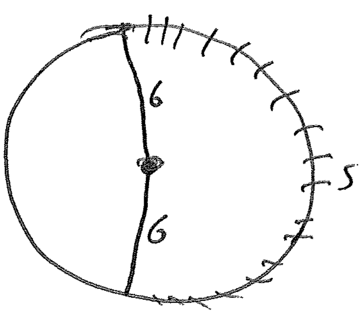
The refrence angle is

$$225^\circ - 180^\circ = 45^\circ$$

4 (10 points) A circle has a radius 6 feet. Find the length of the arc intercepted by a central angle of 240°.

First, we need to switch to radians:

$$240^\circ \cdot \frac{\pi \text{ radians}}{180^\circ} = \frac{4\pi}{3} \text{ radians.}$$



$$s = r \cdot \theta = 6 \cdot \frac{4\pi}{3} = 8\pi$$

is the length.

$\boxed{5}$ (30 points)

(a) (10 points) If $\cos(\theta) = \frac{\sqrt{6}}{3}$, and θ is an acute angle, then find $\csc(\theta)$.

$\csc(\theta) = \frac{1}{\sin\theta}$

we need to find $\sin(\theta)$ first.

$\cos^2(\theta) + \sin^2(\theta) = 1 \qquad \left(\frac{\sqrt{6}}{3}\right)^2 + \sin^2(\theta) = 1$

$\sin^2(\theta) = 1 - \frac{6}{9} = \frac{3}{9} = \frac{1}{3} \implies \sin(\theta) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Therefore, $\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{1/\sqrt{3}} = \sqrt{3}$

(b) (10 points) Find exact value of $\csc(35°)\sin(35°) + \sec(15°)\cos(15°)$.

$$= \cancel{\frac{1}{\sin(35°)}} \cdot \cancel{\sin(35°)} + \cancel{\frac{1}{\cos(15°)}} \cdot \cancel{\cos(15°)}$$

$$= 1 + 1 = 2.$$

(c) (10 points) Find the exact value of

$$\sin 15° \cos 75° + \frac{\cos 15°}{\sec 15°}$$

$\sin 15° \xrightarrow{\text{cofunction}} \cos(75°)$, $\quad \sec 15° \to \frac{1}{\cos(15°)}$

$$= \cos(75°) \cdot \cos(75°) + \frac{\cos(15°)}{\frac{1}{\cos(15°)}}$$

$$= \cos^2(75°) + \cos(15°) \cdot \cos(15°)$$

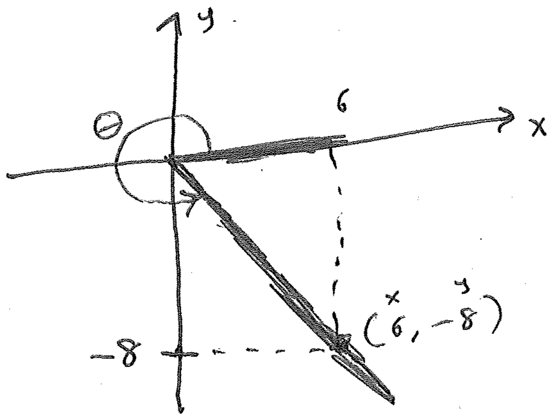
$$= \cos^2(75°) + \cos^2(15°)$$

$$= \cos^2(75°) + \sin^2(75°) \quad \text{by cofunctions}$$

$$= 1 \quad \text{by pythogorean.}$$

6 (15 points)

(a) (10 points) If the point $(6, -8)$ is on the terminal side of an angle θ, then find the exact values of $\cot(\theta)$ and $\csc(\theta)$.

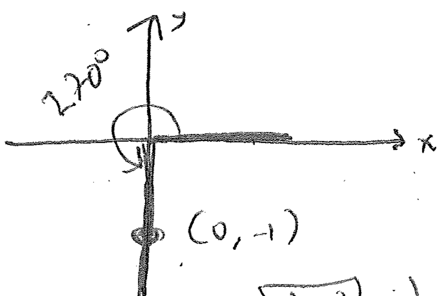


$$\cot(\theta) = \frac{x}{y} = \frac{6}{-8} = -\frac{3}{4}$$

$$\csc(\theta) = \frac{r}{y} = \frac{10}{-8} = -\frac{5}{4}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{6^2 + (-8)^2} = 10$$

(b) (5 points) Evaluate $\sin(270°)$. Show your work.



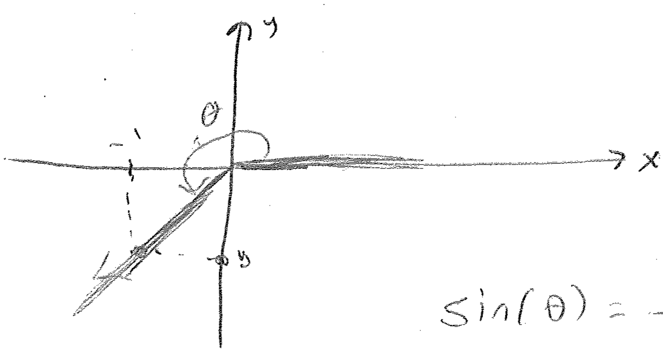
$r = \sqrt{0^2 + 1^2} = 1$

We pick any point on the terminal side: for example $(0, -1)$

$$\sin(270°) = \frac{y}{r} = \frac{-1}{1} = -1.$$

7 (10 points) If $\sec(\theta) = -3$, and $\tan(\theta) > 0$, then find the exact values of $\sin\theta$.

Tangent is positive only on the first and third quadrant. But since $\sec(\theta)$ is negative, the angle has to be on the third quadrant. Remember $\sec(\theta)$ and $\cos(\theta)$ have the same sign.



$$\sec(\theta) = \frac{r}{x} = -3 \Rightarrow r = 3,\ x = -1$$

$$r = \sqrt{x^2 + y^2} \Rightarrow 3 = \sqrt{(-1)^2 + y^2}$$

$$9 = 1 + y^2$$

$$y^2 = 8 \Rightarrow y = -\sqrt{8}$$

we pick the negative b/c

$$\sin(\theta) = \frac{y}{r} = \frac{-\sqrt{8}}{3} = -\frac{\sqrt{8}}{3} = -\frac{2\sqrt{2}}{3}$$