MAC 1114 — # EXAM TWO — FALL-2016

Name

Signature

Problem	Total Points	Score
1	20	
2	20	
3	10	
4	10	
5	20	
6	10	
7	20	
Total	110	

- You are not permitted to use a calculator on this exam.
- In order to receive full credit, you must show your work. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Good luck!

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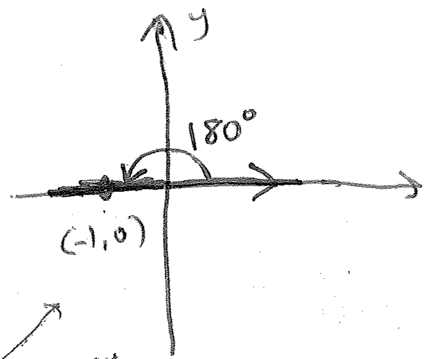
$\boxed{1}$ (20 points)

(a) (5 points) Find the exact value of $\tan(180°)$.



we pick any point on the terminal side.

$$\tan(180°) = \frac{y}{x} = \frac{0}{-1} = 0$$

(b) (5 points) Find the value of $4\sin(33°)\csc(33°) - 7\sec(58°)\cos(58°)$. Use reciprocal identities

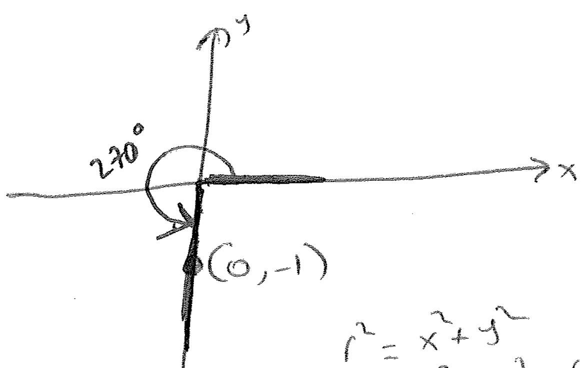
$$= 4\cancel{\sin(33°)} \cdot \frac{1}{\cancel{\sin(33°)}} - 7 \cdot \cancel{\sec(58°)} \cdot \frac{1}{\cancel{\sec(58°)}}$$

$$= 4 \cdot 1 - 7 \cdot 1 = -3$$

(c) (10 points) ~~Use the reference angle to~~ find the exact value of $\sin(990°)$.

$$\sin(990°) = \sin(720° + 270°) = \sin(270°)$$

because sine is periodic with period 360°.



$r^2 = x^2 + y^2$

$r^2 = 0^2 + (-1)^2 = 1$

$$\sin(270°) = \frac{y}{r} = \frac{-1}{1} = -1$$

Thus, $\sin(990°) = -1$

2 (20 points) Determine the amplitude, period, and phase shift of $y = \frac{1}{2}\sin(\pi x + \pi)$. Then graph the function (you should graph the function for more than one period).

$A = \frac{1}{2} \quad B = \pi, \quad C = -\pi$

$\text{Amplitude} = |A| = |\frac{1}{2}| = \frac{1}{2}$

$\text{period} = \frac{2\pi}{B} = \frac{2\cancel{\pi}}{\cancel{\pi}} = 2$

$\text{phase shift} = \frac{C}{B} = \frac{\cancel{-\pi}}{\cancel{\pi}} = -1$

Now, the key points:

$X_1 = \frac{C}{B} = -1$

$y_1 = \frac{1}{2}\sin(-\pi+\pi) = \frac{1}{2}\sin(0) = 0$

$X_2 = X_1 + \frac{\text{period}}{4} = -1 + \frac{2}{4} = -\frac{1}{2}$

$y_2 = \frac{1}{2}(\sin(-\frac{\pi}{2} + \pi)) = \frac{1}{2}\sin(\frac{\pi}{2}) = \frac{1}{2}$

$X_3 = X_2 + \frac{\text{period}}{4} = -\frac{1}{2} + \frac{2}{4} = 0$

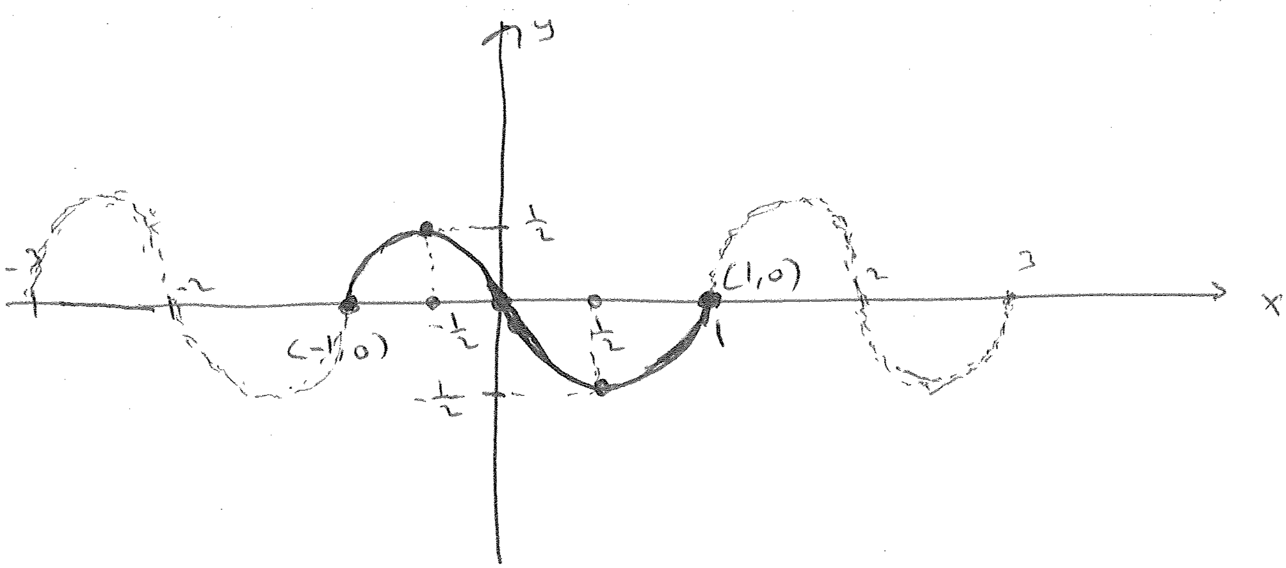
$y_3 = \frac{1}{2}\sin(0 + \pi) = \frac{1}{2}\cdot 0 = 0$

$X_4 = X_3 + \frac{\text{period}}{4} = 0 + \frac{2}{4} = \frac{1}{2}$

$y_4 = \frac{1}{2}\sin(\frac{\pi}{2} + \pi) = \frac{1}{2}\sin(\frac{3\pi}{2}) = \frac{1}{2}\cdot -1 = -\frac{1}{2}$

$X_5 = X_4 + \frac{\text{period}}{4} = \frac{1}{2} + \frac{2}{4} = \frac{1}{2} + \frac{1}{2} = 1$

$y_5 = \frac{1}{2}\sin(\pi + \pi) = \frac{1}{2}(\sin 2\pi) = \frac{1}{2}\cdot 0 = 0.$



$\boxed{3}$ (10 points)

(a) (5 points) Write an equation of a cosine function with amplitude is 4, period is $\frac{\pi}{3}$, and phase shift is $\frac{1}{2}$.

$$y = 4 \cdot \cos(6x - 3)$$

because then $|A| = 4$, $B = 6$, and $C = 3$

So, amplitude is $|A| = 4$, period $= \frac{2\pi}{B} = \frac{2\pi}{6} = \frac{\pi}{3}$ and phase shift $= \frac{C}{B} = \frac{3}{6} = \frac{1}{2}$.

(b) (5 points) Fnd the asymptotes of $y = 2\csc(2x)$ on the interval $[0, \pi]$.

$y = 2\csc(2x) = \frac{2}{\sin(2x)}$ so, whereever $\sin(2x) = 0$ we have an asymptote.

$\sin(2x) = 0$ for $x = 0$, $x = \frac{\pi}{2}$, and $x = \pi$

b/c $\sin(0) = 0$, $\sin(\pi) = 0$ and $\sin(2\pi) = 0$.

Thus, $x = 0$, $x = \frac{\pi}{2}$, and $x = \pi$ are the asymptotes.

$\boxed{4}$ (10 points)

(a) (5 points) What is the domain of tangent?

$\tan(t) = \frac{\sin(t)}{\cos(t)}$, so, the domain is all real numbers except where $\cos(t)$ is 0.

And, cosine is 0 for the angles those are odd multiples of $\frac{\pi}{2}$.

Thus, domain is all real numbers except odd multiples of $\frac{\pi}{2}$.

(b) (5 points) Why do we need to do restrictions on the domains of trigonometric functions to define inverse trigonometric functions?

Because otherwise they don't pass the Horizontal Line Test and they don't have an inverse.

5 (20 points) Graph the function $y = 2\cot(x - \frac{\pi}{2})$. $Bx - C$

(1) We set $Bx-C=0$ and $Bx-C=\pi$ to find the asymptotes.

$$x-\frac{\pi}{2}=0 \quad \text{and} \quad x-\frac{\pi}{2}=\pi$$

$$x=\frac{\pi}{2} \quad \text{and} \quad x=\frac{3\pi}{2}$$

(2) the mid point of this two points will be our x-int.

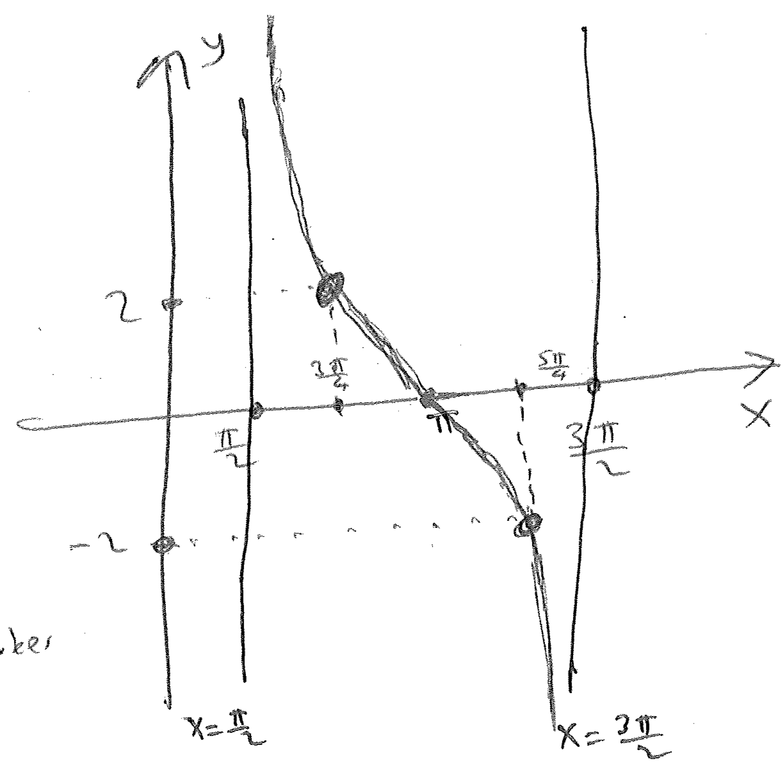
$$\text{mid point} = \frac{\frac{\pi}{2}+\frac{3\pi}{2}}{2} = \frac{\frac{4\pi}{2}}{2} = \frac{4\pi}{4} = \pi$$

(3) $\frac{1}{4}$ and $\frac{3}{4}$ between these two asymptotes are

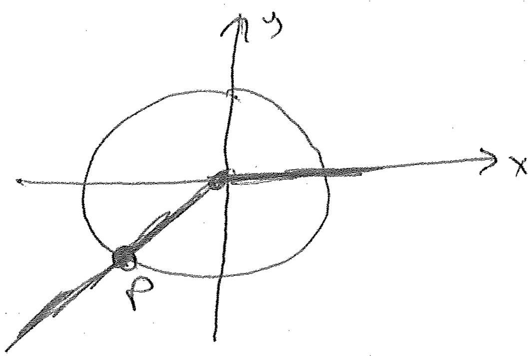
$$\frac{\frac{\pi}{2}+\pi}{2} = \frac{\frac{3\pi}{2}}{2} = \frac{3\pi}{4}$$

$$\frac{\pi+\frac{3\pi}{2}}{2} = \frac{\frac{5\pi}{2}}{2} = \frac{5\pi}{4}$$

So, at $\frac{3\pi}{4}$ the function takes value 2, and at $\frac{5\pi}{4}$ it takes the value -2.



6 (10 points) The point $P = (-\frac{5}{13}, -\frac{12}{13})$ is a point on the unit circle corresponding to real number t. Find the exact value of $\tan(t)$.



$$\tan(t) = \frac{y}{x} = \frac{-12/13}{-5/13} = \left(-\frac{12}{\cancel{13}}\right) \cdot \left(-\frac{\cancel{13}}{5}\right)$$

$$= \frac{-12}{-5} = \frac{12}{5}.$$

$$\tan(t) = \frac{12}{5}$$

7 (20 points)

(a) (10 points) Find the exact value of $\sin^{-1}(-\frac{\sqrt{3}}{2})$.

Let $\sin^{-1}(-\frac{\sqrt{3}}{2}) = \theta$, so we are looking for θ.

this means $\sin(\theta) = -\frac{\sqrt{3}}{2}$ where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ (restricted domain)

So, when sine is $-\frac{\sqrt{3}}{2}$?

θ has to be in quadrant IV b/c $\sin(\theta) = -\frac{\sqrt{3}}{2}$ is negative.

$\sin(-\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$, therefore $\theta = -\frac{\pi}{3}$.

(b) (10 points) Find the exact value of $\cos^{-1}(-\frac{1}{2})$.

Let $\theta = \cos^{-1}(-\frac{1}{2})$, so we are looking for θ.

means $\cos(\theta) = -\frac{1}{2}$ where $\theta \in [0, \pi]$

θ has to be in the second quadrant because cosine is negative.

θ has to be $120^\circ = \frac{2\pi}{3}$ because $\cos(\frac{2\pi}{3}) = -\frac{1}{2}$

and $\frac{2\pi}{3} \in [0, \pi]$.

$$\theta = \frac{2\pi}{3}$$