Note: The actual exam will be shorter.

MAC 1114 EXAM 4

$\boxed{1}$

(a) (5 points) Verify the identity

$$\cos\theta \cdot \csc\theta \cdot \tan\theta = 1$$

The left hand side is

$$= \cos(\theta) \cdot \frac{1}{\cancel{\sin(\theta)}} \cdot \frac{\cancel{\sin(\theta)}}{\cos(\theta)}$$

$$= \cancel{\cos(\theta)} \cdot \frac{1}{\cancel{\cos(\theta)}} = 1 \checkmark$$

(b) (10 points) Verify the identity

$$\frac{\overbrace{\sec(2\theta)}^{\frac{1}{\cos(2\theta)}} - \cos(2\theta)}{\sin^2(2\theta)} = \sec(2\theta)$$

From the left hand side:

$$= \frac{\frac{1}{\cos(2\theta)} - \cos(2\theta)}{\sin^2(2\theta)} = \frac{\frac{1-\cos^2(2\theta)}{\cos(2\theta)}}{\sin^2(2\theta)}$$

$$= \frac{\cancel{(1-\cos^2(2\theta))}}{\cos(2\theta)} \cdot \frac{1}{\cancel{\sin^2(2\theta)}} = \frac{1}{\cos(2\theta)}$$

$= \sec(2\theta)$ because $\sin^2(2\theta) = 1 - \cos^2(2\theta)$ by Pythagorean.

(c) (4 points) Find the exact value of $\sin^{-1}(\sin(\frac{5\pi}{6}))$. Explain your answer.

Since $\frac{5\pi}{6}$ is not in $(-\frac{\pi}{2}, \frac{\pi}{2})$, the answer is NOT $\frac{5\pi}{6}$.

$\sin(\frac{5\pi}{6}) = \sin(\frac{\pi}{6}) = \frac{1}{2}$ by the reference angle idea.

$\sin^{-1}(\sin(\frac{5\pi}{6})) = \sin^{-1}(\sin\frac{\pi}{6}) = \frac{\pi}{6}$ because $\frac{\pi}{6}$ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

(d) (5 points) Find the exact value of $\tan(\sin^{-1}(-\frac{4}{5}))$.

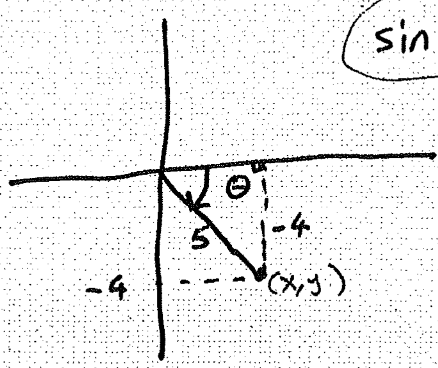
Let $\theta = \sin^{-1}(-\frac{4}{5})$, so we are looking for $\tan(\theta)$!!

$\downarrow$

this means $\sin(\theta) = -\frac{4}{5}$ and $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

but sine is negative θ has to be in $[-\frac{\pi}{2}, 0]$ interval.

$\sin(\theta) = \frac{y}{r} = \frac{-4}{5}$



Therefore, by Pythogeran theorem $x = 3$

$\Rightarrow$ and, so $\tan(\theta) = \frac{y}{x} = \frac{-4}{3} = -\frac{4}{3}$

$\Rightarrow \tan(\sin^{-1}(-\frac{4}{5})) = -\frac{4}{3}$

(e) (5 points) Find the exact value of $\sin(75°)$

$\sin(75°) = \sin(30° + 45°)$ by sum formula for sine

$= \sin(30°) \cdot \cos(45°) + \sin(45°) \cdot \cos(30°)$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$.

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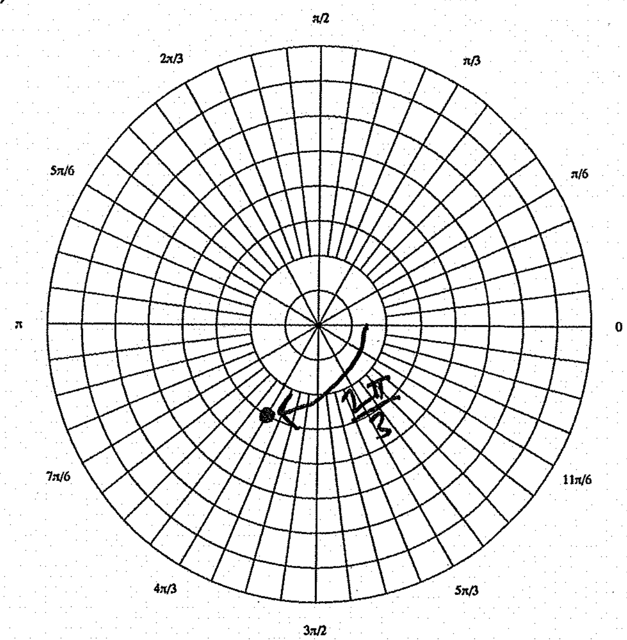
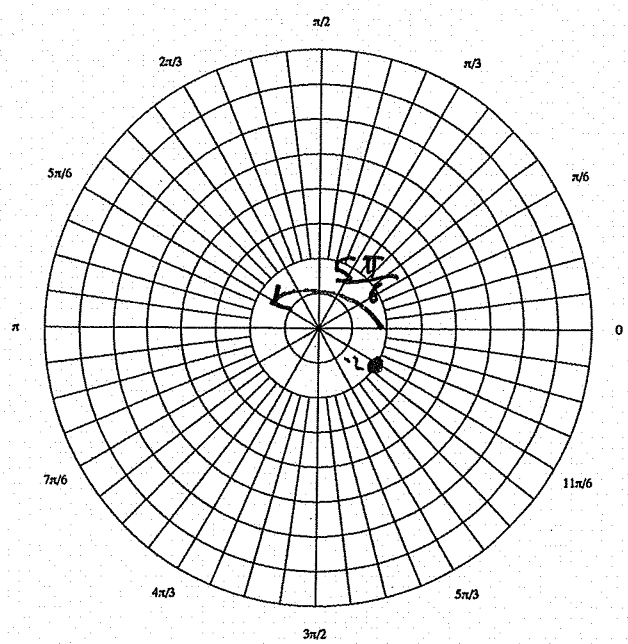
$\boxed{2}$ (10 points) Plot the following points in polar coordinates

(a) (5 points) $\left(3, -\frac{2\pi}{3}\right)$



(b) (5 points) $\left(-2, \frac{5\pi}{6}\right)$



MAC 1114 EXAM 4

$\boxed{3}$ (10 points)

(a) (5 points) Find the rectangular coordinates of $(r, \theta) = (-2, 135°)$.

$x = r\cos\theta$

$x = (-2).\cos(135°)$

$x = (-2).(-\cos(45°))$

$x = (-2).(-\frac{\sqrt{2}}{2}) = \sqrt{2}$

$y = r\sin\theta$

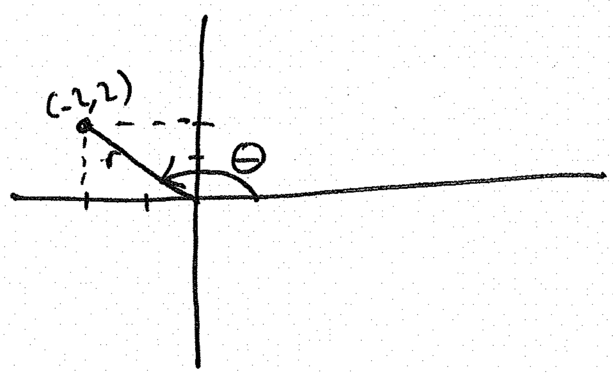
$y = (-2).\sin(135°)$

$y = (-2).\sin(45°)$

$y = -\cancel{2}.\frac{\sqrt{2}}{\cancel{2}} = -\sqrt{2}$

The rectangular coordinates are $(\sqrt{2}, -\sqrt{2})$

(b) (5 points) Find the polar coordinates of $(x, y) = (-2, 2)$ such that $r > 0$ and $0 \le \theta \le 2\pi$.



$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-2)^2 + 2^2} = \sqrt{8}$

$r = 2\sqrt{2}$

$\tan\theta = \frac{y}{x} = \frac{-2}{2} = -1$

$\tan\theta = -1$ but θ is in the quadrant II.

$\theta = 135° = \frac{3\pi}{4}$

The polar coordinates are $(r, \theta) = (2\sqrt{2}, \frac{3\pi}{4})$.

MAC 1114 EXAM 4

$\boxed{4}$ (20 points)

(a) (10 points) Convert the polar equation to a rectangular equation

$$r = 14\sec\theta$$

Since $\sec\theta = \frac{1}{\cos\theta}$, $r = 14\sec\theta$

$r = 14 \cdot \frac{1}{\cos\theta}$ (multiply both sides by $\cos\theta$

$r \cdot \cos\theta = 14$ (since $x = r\cos\theta$

$$\boxed{x = 14}$$

(b) (10 points) Convert the polar equation to a rectangular equation

$$r^2 \sin(2\theta) = 4$$

Since $\sin(2\theta) = 2\sin\theta \cdot \cos\theta$, we have

$r^2 \cdot 2 \cdot \sin\theta \cdot \cos\theta = 4$

$\underbrace{r \cdot \sin\theta}_{y} \cdot \underbrace{r \cdot \cos\theta}_{x} = 2$

$$\boxed{x \cdot y = 2}$$

5 (15 points) Solve the triangle with $a = 5$, $b = 5\sqrt{3}$, and $B = 60°$. You need to use either the law of sines or the law of cosines for this question, other methods will be disregarded.

This is a SSA triangle, which is the ambiguous case.

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{5}{\sin A} = \frac{5\sqrt{3}}{\sin 60} \Rightarrow \sin A = \frac{5 \cdot \frac{\sqrt{3}}{2}}{5\sqrt{3}}$$

So, $\sin A = \frac{1}{2}$. That means A is either $30°$ or $150°$.

But, $150°$ impossible as $150° + 60° > 180°$.

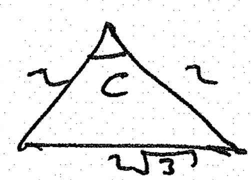
So, $A = 30°$. Then, sine $A+B+C = 180° \Rightarrow \boxed{C = 90°}$

$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{5}{\sin 30°} = \frac{c}{\sin 90°} \Rightarrow c = \frac{5.1}{1/2} = 10$$

$$\Rightarrow \boxed{c = 10}.$$

6 (10 points) Find the largest angle of the triangle with $a = 2$, $b = 2$, and $c = 2\sqrt{3}$. You need to use either the law of sines or the law of cosines for this question, other methods will be disregarded.

The largest angle is opposite to the largest side:



$c^2 = a^2 + b^2 - 2ab\cos C$

$(2\sqrt{3})^2 = 2^2 + 2^2 - 2.2.2.\cos C$

$12 = 8 - 8.\cos C$

$4 = -8.\cos C$

$\cos C = -\frac{1}{2} \Rightarrow \boxed{C = 120°}$

because $\cos 120 = -\cos 60 = -\frac{1}{2}$.

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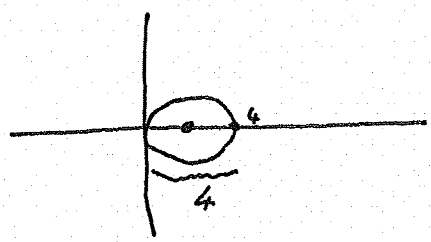
$\boxed{7}$ (10 points) Identify the polar graph (line, circle, cardioid, limacon, rose): If a circle, name the center and the radius. If a limacon, name the type. If a rose, state the number of petals.

(a) (5 points) $r = -5 \sin 3\theta$

This is a rose curve, with 3 petals.

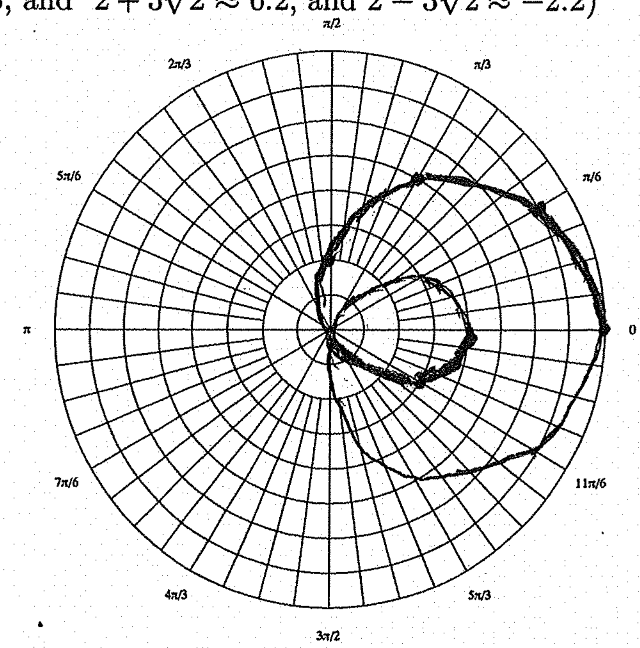
(b) (5 points) $r = 4 \cos \theta$

This is a circle, centered at $(2,0)$ with radius 2.



$\boxed{8}$ (15 points) Test for symmetry and graph the polar equation $r = 2 + 6\cos(\theta)$ (note that $2 + 3\sqrt{3} \approx 7$, and $2 - 3\sqrt{3} \approx -3$, and $2 + 3\sqrt{2} \approx 6.2$, and $2 - 3\sqrt{2} \approx -2.2$)

This is a limaçon.



Polar axis symmetry:

Replace θ by $-\theta$:

$r = 2 + 6 \cdot \cos(-\theta) = 2 + 6 \cdot \cos\theta$ ✓

Yes.

Symmetry w.r.t. line $\theta = \frac{\pi}{2}$:

replace (r, θ) by $(-r, -\theta)$:

$-r = 2 + 6\cos(-\theta)$

$r = -2 - 6\cos(\theta)$. Maybe.

Symmetry w.r.t. the pole:

replace r by $-r$:

$-r = 2 + 6\cos\theta$

$r = -2 - 6\cos\theta$, maybe.

Mak a table:

θ	r
0	$r=8$
$\frac{\pi}{6}$	$r=7$
$\frac{\pi}{3}$	$r=5$
π	$r=2$

θ	r
$\frac{2\pi}{3}$	-1
$\frac{5\pi}{6}$	-3
π	$-$

And, finally, use that the graph is symmetric with respect to

MAC 1114 EXAM 4

$\boxed{9}$ (20 points) Test for symmetry and graph the polar equation

$$r = 8\cos(2\theta)$$

Polar axis symmetry:

replace θ by $-\theta$

$r = 8 \cdot \cos(-2\theta)$

$= 8\cos(2\theta)$ ✓

Yes.

Symmetry w.r.t. line $\theta = \frac{\pi}{2}$:

replace (r, θ) by $(-r, \theta)$:

$-r = 8 \cdot \cos(-2\theta)$

$-r = 8 \cdot \cos(2\theta)$

$r = -8 \cos(2\theta)$

maybe

The pole:

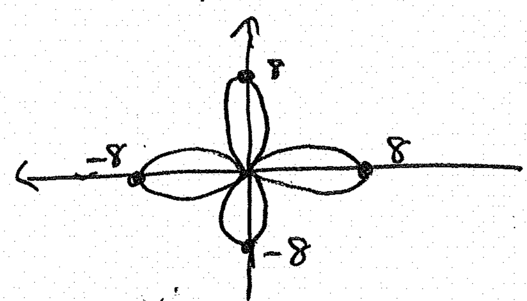
replace r with $-r$:

$-r = 8\cos(2\theta)$

$r = -8\cos(2\theta)$

maybe

This is a rose curve with 4 petals:



$\boxed{10}$ (20 points) Test for symmetry and graph the polar equation

$$r = 3 - 4\sin(\theta)$$

Polar axis symmetry:

replace θ with $-\theta$

$r = 3 - 4(\sin(-\theta))$

$= 3 + 4\sin\theta$

maybe

Symmetry w.r.t. line $\theta = \frac{\pi}{2}$:

replace (r, θ) by $(-r, -\theta)$

$-r = 3 - 4(\sin(-\theta))$

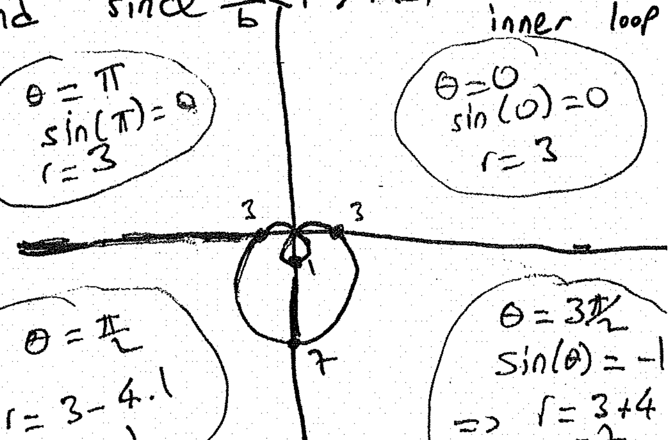
$-r = 3 + 4\sin\theta$

$r = -3 - 4\sin\theta$, so maybe

We know this is a Limaçon and since $\frac{a}{b} < 1$, then it has an inner loop

$\theta = \pi$
$\sin(\pi) = 0$
$r = 3$

$\theta = 0$
$\sin(0) = 0$
$r = 3$



$\theta = \frac{\pi}{2}$

$r = 3 - 4 \cdot 1$
$r = -1$

$\theta = 3\pi/2$
$\sin(\theta) = -1$
$\Rightarrow r = 3 + 4$
$= 7$

The pole:

replace r by $-r$

$-r = 3 - 4\sin(\theta)$

$r = -3 + 4\sin\theta$

so, maybe

MAC 1114 EXAM 4 .

$\boxed{10}$ (10 points)

(a) (5 points) Find the quotient $\frac{z_1}{z_2}$. Leave the answer in polar form.

$$z_1 = 16(\cos 28° + i \sin 28°), \qquad z_2 = 8(\cos 4° + i \sin 4°)$$

$$\frac{z_1}{z_2} = \frac{16}{8}\left(\cos(28° - 4°) + i \sin(28° - 4°)\right)$$

$$= 2(\cos 24° + i \sin 24°)$$

(b) (5 points) Take the power of the following complex number. Leave the answer in polar form.

$$[5(\cos 105° + i \sin 105°)]^3$$

By De Moivre's formula:

$$5^3(\cos(3.105) + i \cdot \sin(3.105))$$

$$125(\cos 315° + i \cdot \sin 315°)$$

$$125(\cos 45° + i \cdot (-\sin 45°))$$

$$125 \cdot \frac{\sqrt{2}}{2} - i \cdot 125 \cdot \frac{\sqrt{2}}{2}.$$

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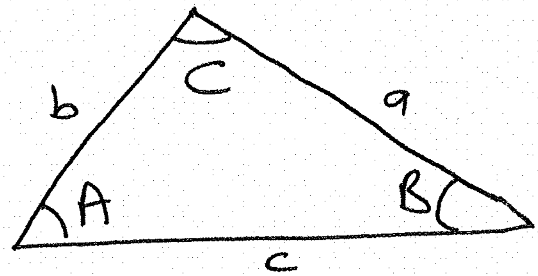
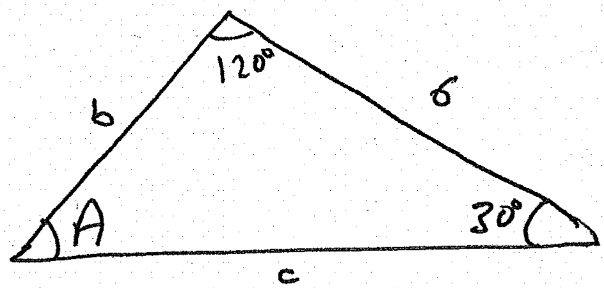
$\boxed{12}$ (15 points) Solve the triangle with $a = 6$, $C = 120°$, and $B = 30°$. You need to use either the law of sines or the law of cosines for this question, other methods will be disregarded.





$A = 180 - 120° - 30° = 30°$. $\Rightarrow$ $\boxed{A = 30°}$

By the law of sines

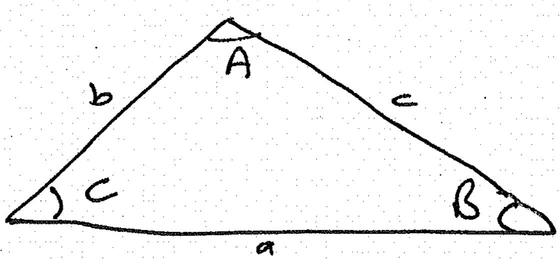
$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{6}{\cancel{\sin 30°}} = \frac{b}{\cancel{\sin 30°}} \Rightarrow \boxed{b = 6}$$

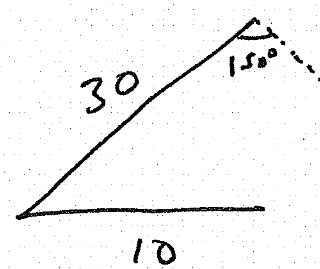
$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{6}{\sin 30°} = \frac{c}{\sin 120°} \Rightarrow c = \frac{6 \cdot \sin 120°}{\sin 30°}$$

$$\Rightarrow c = \frac{6 \cdot \sqrt{3}/2}{1/2}$$

$c = 6\sqrt{3}$

$\boxed{13}$ (15 points) Solve the triangle with $a = 10$, $b = 30$, and $A = 150°$. You need to use either the law of sines or the law of cosines for this question, other methods will be disregarded.





We use the law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{10}{\sin 150} = \frac{30}{\sin B}$$

$$\Rightarrow \sin B = \frac{30 \cdot \sin 150°}{10} = \frac{30 \cdot \sin 30°}{10}$$

$$= \frac{30 \cdot 1/2}{10} = 3/2 > 1$$

But, sin can not be greater than 1. So, there is no solution.