MAC 1114 EXAM ONE SPRING-2016

$\boxed{1}$ (15 points)

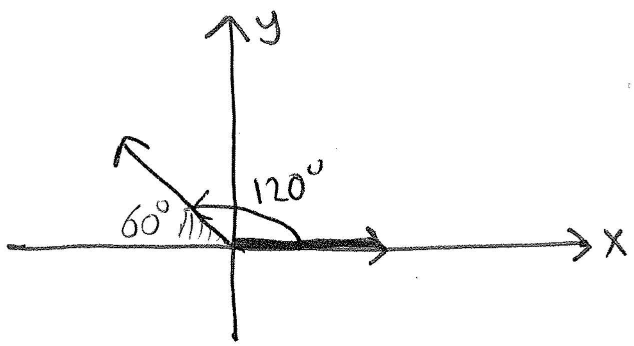
(a) (5 points) Convert $-240°$ to radians.

$$-\cancel{240°} \cdot \frac{\pi}{\cancel{180°}} = -\frac{4\pi}{3}.$$

(b) (5 points) Find a positive angle less than $360°$ that is coterminal with $-240°$.

$-240° + 360° = 120°$ is coterminal with $-240°$ and it is positive and less than $360°$.

(c) (5 points) Find the value of $\cos(-240°)$.



$-240°$ is coterminal with $120°$.

$60°$ is the refrence angle of $120°$.

So, $\cos(-240) = -\cos(60°)$
$= -\frac{1}{2}$.

We put minus sign becouse we know cosine is negative in quadrant II

$\boxed{2}$ (10 points)

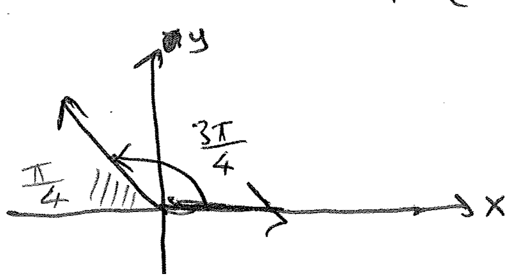
(a) (5 points) Find a positive angle less than 2π that is coterminal with $\frac{19\pi}{4}$.

$\frac{19\pi}{4} - 2\pi = \frac{19\pi}{4} - \frac{8\pi}{4} = \frac{11\pi}{4}$ but that is greater than 2π.

$\frac{11\pi}{4} - 2\pi = \frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$.

(b) (5 points) Find the value of $\sin(\frac{19\pi}{4})$.

$\sin\left(\frac{19\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ (sine is positive in quadrant II –



$\frac{\pi}{4}$ is the refrence angle of $\frac{3\pi}{4}$.

$\boxed{3}$ (15 points)

(a) (5 points) Find the value of $\sin^2(10°) + \sin^2(80°) - 1. = 0$

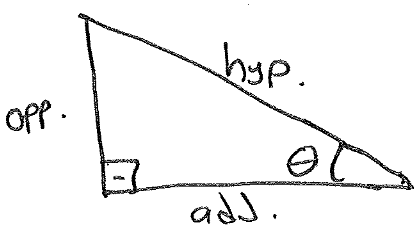
Use cofunctions idea:

$$\sin(80) = \cos(10)$$
$$\sin^2(80) = \cos^2(10)$$

$$\Rightarrow \quad \sin^2(10) + \sin^2(80) = \sin^2(10) + \cos^2(10) = 1$$

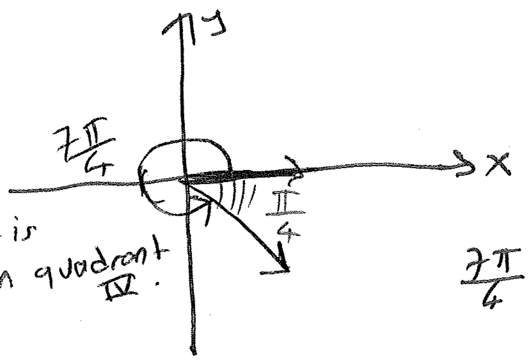
by Pythagorean.

(b) (5 points) If $\sin = \frac{\text{opposite}}{\text{hypotenuse}}$, explain why sine can never be greater than one.



Sine has to be less than 1 because the length of the opposide side has to be less than the length of the hypotenuse.

(c) (5 points) Use the reference angle to find the exact value of $\cot(-\frac{\pi}{4})$.



cotangent is negative in quadrant IV.

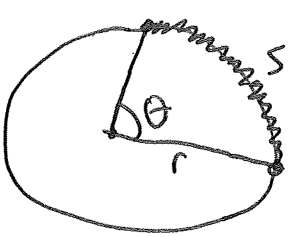
$$-\frac{\pi}{4} + 2\pi = -\frac{\pi}{4} + \frac{8\pi}{4} = \frac{7\pi}{4}$$

So, $\frac{7\pi}{4}$ and $-\frac{\pi}{4}$ are coterminal angles.

$\frac{7\pi}{4}$ and $\frac{\pi}{4}$ are refrence angles;

$$\cot(-\tfrac{\pi}{4}) = -\cot(\tfrac{\pi}{4}) = -\frac{\cos(\frac{\pi}{4})}{\sin(\frac{\pi}{4})} = -1$$

$\boxed{4}$ (10 points) A circle has a radius 8 feet. Find the length of the arc intercepted by a central angle of 270°.



$S = r \cdot \theta$ θ has to be in radians.

$$270° \cdot \frac{\pi}{180°} = \frac{3\pi}{2} \text{ radians.}$$

$$S = 8 \cdot \frac{3\pi}{2} = 12\pi$$ is the length.

$\boxed{5}$ (20 points)

(a) (5 points) If $\sin(\theta) = \frac{\sqrt{3}}{3}$, and θ is an acute angle, then find $\tan(\theta)$.

$$\sin^2(\theta) + \cos^2(\theta) = 1 \implies \cos^2(\theta) = 1 - \left(\frac{\sqrt{3}}{3}\right)^2$$

$$= 1 - \frac{3}{9}$$

$$= \frac{6}{9}$$

$$\cos(\theta) = \frac{\sqrt{6}}{3}.$$

$$\tan\theta = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\sqrt{3}/3}{\sqrt{6}/3} = \frac{\sqrt{3}}{\sqrt{6}} = \sqrt{\frac{1}{2}}.$$

(b) (5 points) Find exact value of $\sec(35°)\cos(35°)$. $= 1$

$$\frac{1}{\cos(35°)} \cdot \cos(35°) = 1$$

becouse $\sec(\theta) = \frac{1}{\cos(\theta)}$;

(c) (5 points) Find exact value of $\sec(60°)\tan(45°) + \csc(30°)$. $= 4$

$$= \frac{1}{\cos(60)} \cdot \frac{\sin(45°)}{\cos(45°)} + \frac{1}{\sin(30°)}$$

$$= \frac{1}{1/2} \cdot 1 + \frac{1}{1/2} = 2 + 2 = 4$$

(d) (5 points) Find the exact value of $\csc(40°)\sec(50°) - \tan(50°)\cot(40°)$.

By cofunctions idea: $\sec(50°) \cdot \sec(50°) - \tan(50°) \cdot \tan(50°)$

$$= \sec^2(50°) - \tan^2(50°)$$

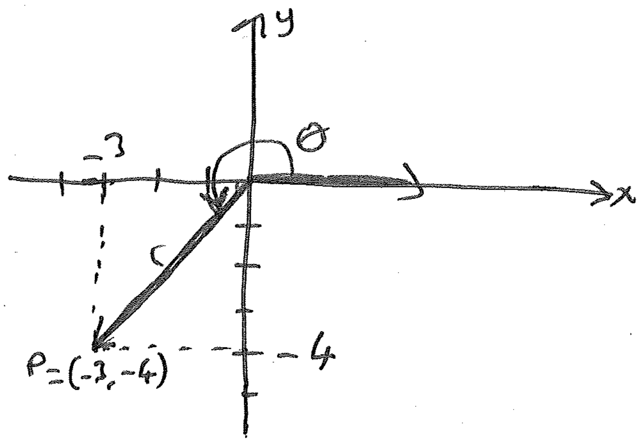
$$= 1$$

because $\sec^2(\theta) = 1 + \tan^2(\theta)$.

$\boxed{6}$ (10 points) If the point $(-3,-4)$ is on the terminal side of an angle θ, then find the exact values of $\cot(\theta)$ and $\sec(\theta)$.

$\frac{\cos(\theta)}{\sin(\theta)}$

$$\cot(\theta) = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$$



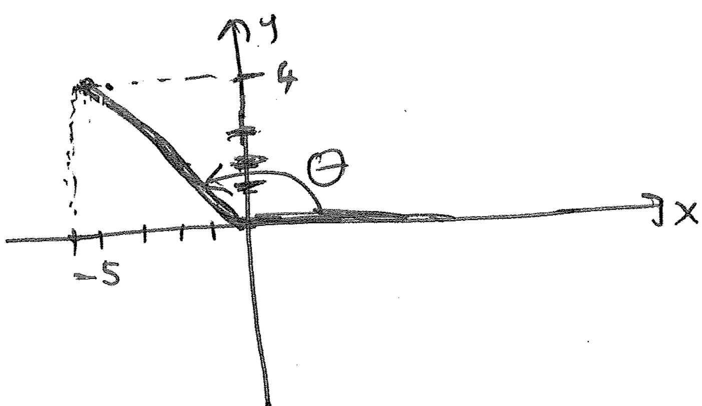
$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{\frac{x}{r}} = \frac{r}{x}$$

$$= \frac{5}{-3} = -\frac{5}{3}$$

$$r = \sqrt{x^2+y^2}$$
$$r = \sqrt{(-3)^2+(-4)^2}$$
$$= \sqrt{25} = 5.$$

$\boxed{7}$ (10 points) If $\tan(\theta) = -\frac{4}{5}$, and $\cos(\theta) < 0$, then find the exact values of $\cos(\theta)$ and $\csc(\theta)$.

Since $\cos(\theta) < 0$, this angle could be either in second or in third quadrant, but in third quadrant tangent is positive. So, θ is in Quadrant II.



Remember $\tan(\theta) = \frac{y}{x}$

So, $\tan(\theta) = -\frac{4}{5} = \frac{y}{x}$

means $y=4$ and $x=-5$ as we are in second quadrant.

$$r = \sqrt{x^2+y^2}$$
$$= \sqrt{(-4)^2+5^2}$$
$$= \sqrt{16+25}$$
$$= \sqrt{41}$$

$$\cos(\theta) = \frac{x}{r} = \frac{-5}{\sqrt{41}} = -\frac{5\sqrt{41}}{41}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{y/r} = \frac{r}{y} = \frac{\sqrt{41}}{4}$$