Solutions.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Total Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>105</td>
<td></td>
</tr>
</tbody>
</table>

- You are not permitted to use a calculator on this exam.
- In order to receive full credit, you must show your work. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Good luck!
EXAM ONE

1 (15 points)

(a) (5 points) Circle the correct one:
Reference angle of an angle is always an **acute** / **obtuse** / **straight** / **quadrantal** angle.

(b) (10 points) A surveyor measured the angle of elevation to be 60°. The transit is 3 feet above the ground (that is the distance between B and D in the graph) and 100 feet from the tower. Find the height of the tower.

\[
\tan 60^\circ = \frac{a}{100}
\]

\[
a = 100 \cdot \tan 60^\circ = 100 \cdot \sqrt{3}
\]

So, the height is

\[
100 \cdot \sqrt{3} + 3
\]

2 (10 points)

(a) (5 points) Find the value of \( \cos^2(15^\circ) + \cos^2(75^\circ) - 3 \).

By co-function idea, \( \cos^2(15^\circ) = \sin^2(75^\circ) \)

So, it is

\[
\sin^2(75^\circ) + \cos^2(75^\circ) - 3 = 1 - 3 = -2
\]

(b) (5 points) Find the value of \( \cos(30^\circ) + \sin(\frac{\pi}{6}) + \tan\left(\frac{\pi}{4}\right) \).

\[
\sqrt{\frac{3}{2}} + \frac{1}{2} + 1 = \frac{3 + \sqrt{3}}{2}
\]

\[
\tan\left(\frac{\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\sqrt{1}/2}{\sqrt{1}/2} = 1
\]
EXAM ONE

3 (15 points)

(a) (5 points) Convert $-\frac{3\pi}{4}$ to degrees.

\[-\frac{3\pi}{4} \cdot \frac{180^\circ}{\pi} = -135^\circ\]

(b) (5 points) Find a positive angle less than 360° that is coterminal with $-135^\circ$.

\[-135^\circ + 360^\circ = 225^\circ\]

(c) (5 points) Find the reference angle of $-135^\circ$.

The reference angle is $225^\circ - 180^\circ = 45^\circ$

4 (10 points) A circle has a radius 6 feet. Find the length of the arc intercepted by a central angle of 240°.

First, we need to switch to radians:

\[240^\circ \cdot \frac{\pi \text{ radians}}{180^\circ} = \frac{4\pi}{3} \text{ radians}\]

\[s = r \cdot \theta = 6 \cdot \frac{4\pi}{3} = 8\pi\]

is the length.
(a) (10 points) If \( \cos(\theta) = \frac{\sqrt{6}}{3} \), and \( \theta \) is an acute angle, then find \( \csc(\theta) \).

we need to find \( \sin(\theta) \) first.

\[
\cos^2(\theta) + \sin^2(\theta) = 1 \quad \left( \frac{\sqrt{6}}{3} \right)^2 + \sin^2(\theta) = 1
\]

\[
\sin^2(\theta) = 1 - \frac{6}{9} = \frac{3}{9} = \frac{1}{3} \quad \Rightarrow \quad \sin(\theta) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}
\]

Therefore, \( \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{\frac{\sqrt{3}}{3}} = \sqrt{3} \)

(b) (10 points) Find exact value of \( \csc(35^\circ) \sin(35^\circ) + \sec(15^\circ) \cos(15^\circ) \).

\[
= \frac{1}{\sin(35^\circ)} \cdot \sin(35^\circ) + \frac{1}{\cos(15^\circ)} \cdot \cos(15^\circ)
\]

\[
= 1 + 1 = 2.
\]

(c) (10 points) Find the exact value of

\[
\sin 15^\circ \cos 75^\circ + \frac{\cos 15^\circ}{\sec 15^\circ} \quad \text{cofunction}
\]

\[
= \cos(75^\circ), \cos(75^\circ) + \frac{\cos(15^\circ)}{\cos(15^\circ)}
\]

\[
= \cos^2(75^\circ) + \cos(15^\circ) \cdot \cos(15^\circ)
\]

\[
= \cos^2(75^\circ) + \cos^2(15^\circ)
\]

\[
= \cos^2(75^\circ) + \sin^2(75^\circ) \quad \text{by cofunctions}
\]

\[
= 1 \quad \text{by pythagorean}
\]
MAC 1114  

EXAM ONE  

FALL 2016

6 (15 points)

(a) (10 points) If the point (6, −8) is on the terminal side of an angle \( \theta \), then find the exact values of \( \cot(\theta) \) and \( \csc(\theta) \).

\[
\cot(\theta) = \frac{x}{y} = \frac{6}{-8} = -\frac{3}{4}
\]

\[
\csc(\theta) = \frac{r}{y} = \frac{10}{-8} = -\frac{5}{4}
\]

\[
r = \sqrt{x^2 + y^2} = \sqrt{6^2 + (-8)^2} = 10
\]

(b) (5 points) Evaluate \( \sin(270^\circ) \). Show your work.

We pick any point on the terminal side: e.g. for example (0, -1)

\[
\sin(270^\circ) = \frac{y}{r} = \frac{-1}{1} = -1
\]

7 (10 points) If \( \sec(\theta) = -3 \), and \( \tan(\theta) > 0 \), then find the exact values of \( \sin(\theta) \).

Tangent is positive only on the first and third quadrant.
But since \( \sec(\theta) \) is negative, the angle has to be on the third quadrant. Remember \( \sec(\theta) \) and \( \cos(\theta) \) have the same sign.

\[
\sec(\theta) = \frac{r}{x} = -3 \implies r = 3
\]

\[
x = \sqrt{x^2 + y^2} \implies 3 = \sqrt{(-1)^2 + y^2}
\]

\[
y = 1 + y^2 \implies 3 = \sqrt{8}
\]

\[
y = 8 = 2\sqrt{2}
\]

\[
\sin(\theta) = \frac{y}{r} = \frac{-\sqrt{8}}{3} = \frac{-2\sqrt{2}}{3}
\]