EXAM THREE

$\boxed{1}$ (10 points) Find the exact value of

$$-3\sin(-20°)\sec(70°)$$

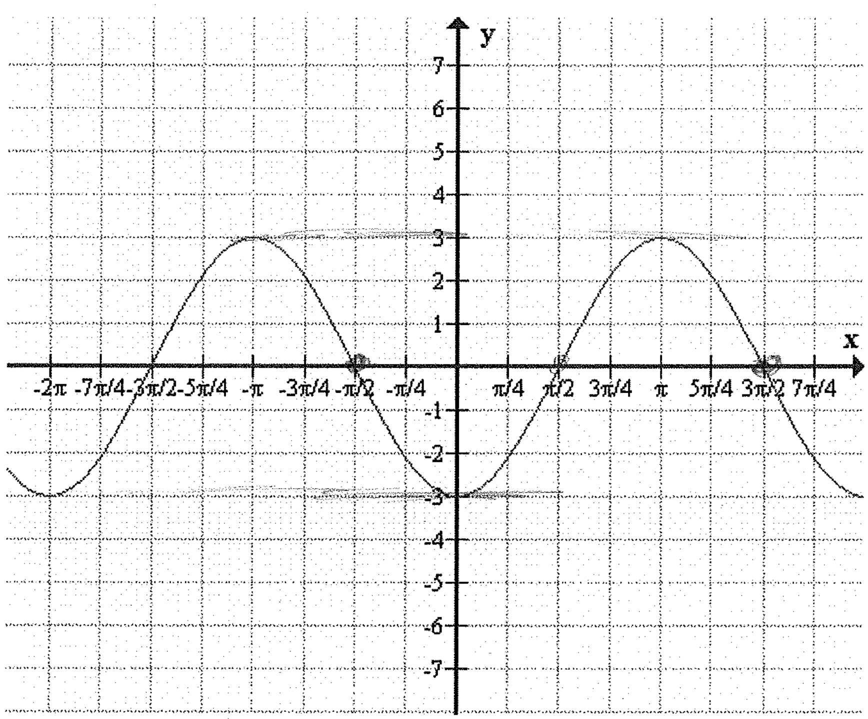
$= -3.(-\sin(20°)).\sec(70°)$ b/c sin is odd

$= 3.\sin(20°).\sec(70°)$

$= 3.\cos(70°).\frac{1}{\cos(70°)} = 3$

b/c $\sin(20°) = \cos(70°)$ and $\sec(70°) = \frac{1}{\cos 70°}$.

$\boxed{2}$ (10 points) Find the equation of the function which has the following graph. Indicate the amplitude, period, and phase shift .



$|A| = 3$ amplitude

phase shift $= \frac{\pi}{2}$

period $= 2\pi$

$\frac{3\pi}{2} - (-\frac{\pi}{2}) = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

so, the period 2π.

$y = A\sin(Bx - C) = 3.\sin(x - \frac{\pi}{2})$

or if you think the graph as a graph of cosine, then

or $y = 3.\cos(x - \pi)$

$\boxed{3}$ (25 points)

(a) (10 points) Verify the identity

$$\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$$

We start with the left hand side

$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \sin\beta \cdot \cos\alpha$ by the sum formula;

$$\frac{\sin\alpha \cdot \cos\beta + \sin\beta \cdot \cos\alpha}{\cos\alpha \cdot \cos\beta} = \frac{\sin\alpha \cdot \cancel{\cos\beta}}{\cos\alpha \cdot \cancel{\cos\beta}} + \frac{\sin\beta \cdot \cancel{\cos\alpha}}{\cancel{\cos\alpha} \cdot \cos\beta}$$

$$= \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \tan\alpha + \tan\beta$$

which is the right hand side

(b) (15 points) Verify the identity

$$\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$$

We start with the left hand side:

$$\frac{1}{1-\sin(\theta)} + \frac{1}{1+\sin\theta} = \frac{1+\sin\theta}{(1-\sin\theta)\cdot(1+\sin\theta)} + \frac{1-\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$(1+\sin\theta)$ $(1-\sin\theta)$

$$= \frac{1+\cancel{\sin\theta}+1-\cancel{\sin\theta}}{(1-\sin\theta)\cdot(1+\sin\theta)} = \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} = 2\sec^2\theta$$

which is the right hand side.

Note that $\cos^2\theta = 1-\sin^2\theta$ by the pythagorean identity.

MAC 1114 EXAM THREE FALL-2016

$\boxed{4}$ (20 points)

(a) (10 points) Find the exact value of $\cos^2(15°) - \sin^2(15°)$

This is the double angle formula for cosine

$$\cos^2(15°) - \sin^2(15°) = \cos(2.(15°)) = \cos(30°) = \frac{\sqrt{3}}{2}$$

This can also be solved by finding $\cos(15°)$ and $\sin(15°)$ individually.

(b) (10 points) Find the exact value of $\sin(75°)$

$$\sin(75°) = \sin(45°+30°) = \sin(45°).\cos(30°) + \cos(45°).\sin(30°)$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$\boxed{5}$ (5 points) What is the difference between "Verifying an Identity" and "Solving an Equation" ?

Verifying an identity means to show that an equation is true for all values of a variable.

Solving an equation is finding a value or values of the variable that make the equation is true.

Verifying an identity is done by rewriting one side of the equation, by using algebra or fundamental identities, to obtain the other side.

6 (15 points)

(a) (5 points) Find the exact value of $\cos(\cos^{-1}(-3))$

$\cos^{-1}(-3)$ is undefined b/c the range of cosine is $[-1, 1]$.

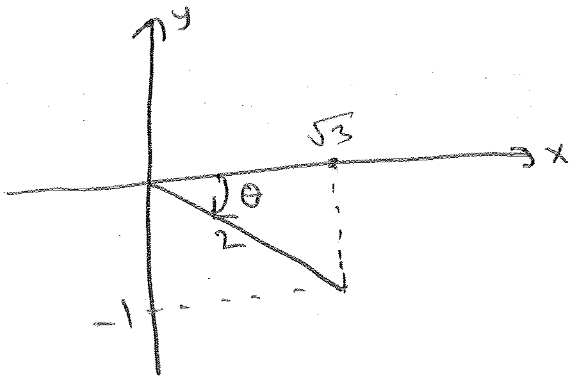
Undefined

(b) (10 points) Find the exact value of $\tan(\underbrace{\sin^{-1}(-\tfrac{1}{2})}_{=\theta})$

Let $\sin^{-1}(-\frac{1}{2})$ be θ ; $\sin^{-1}(-\frac{1}{2}) = \theta$ where $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

$\Rightarrow \sin(\theta) = -\frac{1}{2}$ where $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

Sine is negative in the IV quadrant, so $\theta \in [-\frac{\pi}{2}, 0]$.



$\sin(\theta) = \frac{y}{r} = \frac{-1}{2}$

$\Rightarrow \boxed{y = -1}$ $r = 2$

$r^2 = x^2 + y^2$ $\quad 2^2 = x^2 + (-1)^2$

$x^2 = 3 \Rightarrow \boxed{x = \sqrt{3}}$

Therefore, $\tan(\theta) = \frac{y}{x} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

7 (10 points) Find the exact value of

$$\sin(2\tan^{-1}(-\frac{3}{4})) = \sin(2\theta) = 2\sin\theta.\cos\theta$$

Let $\tan^{-1}(-\frac{3}{4})$ be θ. $\Rightarrow \tan^{-1}(-\frac{3}{4}) = \theta$.
where $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

$\sin(2\tan^{-1}(-\frac{3}{4})) = \sin(2\theta) = 2\sin\theta.\cos\theta$

Since $\tan^{-1}(-\frac{3}{4}) = \theta \Rightarrow \tan(\theta) = -\frac{3}{4} = \frac{y}{x} \Rightarrow y = -3$, $x = 4$
because $\theta \in (-\frac{\pi}{2}, 0)$ since $\tan\theta < 0$.

$r^2 = x^2 + y^2 = 4^2 + (-3)^2 = 25 \Rightarrow r = 5$

So, $\sin(\theta) = \frac{y}{r} = \frac{-3}{5}$ and $\cos(\theta) = \frac{x}{r} = \frac{4}{5}$.

Therefore, $\sin(2\tan^{-1}(-\frac{3}{4})) = \sin(2\theta) = 2\sin\theta.\cos\theta = 2.(-\frac{3}{5}).(\frac{4}{5})$
$= -\frac{24}{25}$.

8 (15 points) Solve the equation over the interval $[0, 2\pi)$

$$\cos(x)\cot(x) = \cos(x)$$

We substract $\cos(x)$ from both sides:

$$\cos(x) \cdot \cot(x) - \cos(x) = 0$$
$$\cos(x) \cdot (\cot(x) - 1) = 0$$

$\Rightarrow$ either $\cos(x) = 0$ or $\cot(x) - 1 = 0$
$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\cot(x) = 1$
$x = \frac{\pi}{4}, \frac{5\pi}{4}$

Solution set $= \{\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}\}$