Florida International University
MAC 1114- Trigonometry
Common Final Exam (Spring 2016)

Name:

PID:

Instructor’s name:

The exam has two parts:

PART I:

Multiple Choice ( 40 points: 20 multiple choice questions worth 2 pts each)

PART II:

SHOW YOUR WORK ( 60 points: 8 questions). All work must be shown to earn full credit. Organize your work and write neatly so it is clear what you do and why.

NO CALCULATOR is allowed.

Turn off and put away all cell phones and other electronic devices

DO NOT PUT YOUR PHONE ON THE CHAIR OR ON THE DESK.
PART I: Multiple Choice.
Mark answers on the scantron sheet using a number 2 pencil.

(1) \( \frac{2\pi}{5} \) radians in degrees is
A) 18°
B) 36°
C) 72°
D) 144°
E) None of these

(2) The reference angle of 250° is
A) 40°
B) 50°
C) 110°
D) 70°
E) 20°

(3) The exact value of \( \cot \left( \sin^{-1} \left( -\frac{1}{2} \right) \right) \) is
A) \(-\sqrt{3}\)
B) \(-1\)
C) \(-\frac{\sqrt{3}}{3}\)
D) \(\sqrt{3}\)
E) \(\frac{\sqrt{3}}{3}\)

(4) \( \cos(2t) \) is equal to
A) \(2 \cos t\)
B) \(\cos^4 t - \sin^4 t\)
C) \(2 \cos t \sin t\)
D) \(1 - \sin^2 t\)
E) None of these
(5) The rectangular equation corresponding to the polar equation \( r = 6 \) is
A) \( xy = 6 \)
B) \( x + y = \pm 6 \)
C) \( x^2 + y^2 = 6 \)
D) \( x^2 + y^2 = 36 \)
E) None of these

(6) The exact value of \( \tan \left( -\frac{14\pi}{3} \right) \) is
A) \( -\sqrt{3} \)
B) \( -\frac{\sqrt{3}}{3} \)
C) \( \frac{\sqrt{3}}{3} \)
D) \( \sqrt{3} \)
E) None of these

(7) From a point that is 300 feet away from the base of a tower, the angle of elevation to the top of the tower is 30°. The height of the tower is
A) 150 feet
B) \( 100\sqrt{3} \) feet
C) \( 150\sqrt{2} \) feet
D) \( 150\sqrt{3} \) feet
E) \( 300\sqrt{3} \) feet

(8) The domain of the function \( y = \cos^{-1} x \) is
A) \( (-\infty, \infty) \)
B) \([0, \pi] \)
C) \( [-\frac{\pi}{2}, \frac{\pi}{2}] \)
D) \([-1, 1] \)
E) None of these
(9) In the triangle $\triangle ABC$, sides $a = 2$, $b = 3$ and angle $\angle C = 60^\circ$. The exact value of side $c$ is
A) $\sqrt{7}$
B) $\sqrt{11}$
C) $\sqrt{13}$
D) $\sqrt{19}$
E) None of these

(10) The length of the arc of a circle with radius 9 feet subtended by a central angle of $100^\circ$ is
A) 900 feet
B) $5\pi$ feet
C) 180 feet
D) $50\pi$ feet
E) None of these

(11) The rectangular equation corresponding to the polar equation $\theta = -\frac{\pi}{6}$ is
A) $y = -\sqrt{3}x$
B) $y = -\frac{1}{2}x$
C) $y = -\frac{\sqrt{3}}{3}x$
D) $y = \frac{\sqrt{3}}{3}x$
E) None of these

(12) The exact value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is
A) $\frac{-\pi}{3}$
B) $\frac{4\pi}{3}$
C) $\frac{5\pi}{6}$
D) $\frac{7\pi}{6}$
E) None of these
(13) Rectangular coordinates \((x, y)\) for the point with polar coordinates \((r, \theta) = \left(4, \frac{5\pi}{6}\right)\) are

A) \((-2, 2)\)
B) \((2, -2\sqrt{3})\)
C) \((-2\sqrt{3}, -2)\)
D) \((-2\sqrt{3}, 2)\)
E) None of these

(14) Find the exact value of \(\sin \left[ \sin^{-1} \frac{3}{5} - \cos^{-1} \frac{-4}{5} \right]\)

A) 0
B) \(\frac{3}{5}\)
C) \(-\frac{24}{25}\)
D) \(-\frac{7}{5}\)
E) None of these

(15) Polar coordinates \((r, \theta)\) of a point having rectangular coordinates \((x, y) = (\sqrt{3}, -1)\) are

A) \(\left(2, -\frac{\pi}{6}\right)\)
B) \(\left(2, -\frac{\pi}{4}\right)\)
C) \(\left(-2, \frac{\pi}{3}\right)\)
D) \(\left(2, \frac{5\pi}{6}\right)\)
E) None of these

(16) If \(\sin \theta = -\frac{1}{3}\) then the exact value of \(\csc \theta\) is

A) \(-3\)
B) \(\frac{3}{\sqrt{8}}\)
C) \(±3\)
D) \(-\frac{3}{\sqrt{8}}\)
E) None of these
(17) \( \frac{\sec^2 x \csc x}{\sec^2 x + \csc^2 x} \) is equal to
A) \( \cos x \)
B) \( \sin x \)
C) \( \csc x \)
D) \( \sec^2 x \csc x \)
E) \( \frac{1}{1 + \csc x} \)

(18) The exact value of \( \cos 25^\circ \cos 5^\circ - \sin 25^\circ \sin 5^\circ \) is
A) 0
B) \( \frac{\sqrt{2}}{2} \)
C) \( \frac{1}{2} \)
D) \( \frac{\sqrt{3}}{2} \)
E) None of these

(19) All solutions of the equation \( \tan(3x) = \sqrt{3} \) are
A) \( \frac{\pi}{3} \)
B) \( \frac{\pi}{18} + \frac{n\pi}{3} \), where \( n \) is any integer
C) \( \frac{\pi}{9} + \frac{n\pi}{3} \), where \( n \) is any integer
D) \( \frac{\pi}{6} + \frac{n\pi}{3} \), where \( n \) is any integer
E) \( \frac{\pi}{3} + \frac{n\pi}{3} \), where \( n \) is any integer

(20) The complex number \( (\cos 5^\circ + i \sin 5^\circ)^9 \) is equal to
A) \( 1 + i \)
B) \( 1 - i \)
C) \( \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \)
D) \( \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \)
E) None of these

END OF PART I
SHOW YOUR WORK (60 points: 8 questions). All work must be shown to earn full credit. Organize your work and write neatly so it is clear what you do and why.

Question 1 [5 points] Find $\sin(2\theta)$ if $\cos \theta = -\frac{4}{5}$ and $\pi \leq \theta \leq \frac{3\pi}{2}$

Question 2 [10 points] If $\cot \theta = -\frac{1}{3}$, and $\sin \theta < 0$, find the exact values of each of the remaining trigonometric functions of $\theta$. (you need to find $\tan \theta$, $\sin \theta$, $\cos \theta$, $\sec \theta$, $\csc \theta$).
Question 3 [8 points] Solve the equation \( \cos(2x) = -\frac{\sqrt{3}}{2} \) on the interval \( 0 \leq x < 2\pi \).

Question 4 [5 points] Write as an algebraic expression (without any trigonometric functions):
\( \cos(\sin^{-1} x - \cos^{-1} y) \)
Question 5 [8 points] Two people stand 500 feet apart. A tree that is perpendicular to the ground is on the line that separates these two people. The angles of elevation from each person to the top of the tree measure 20° and 40°, respectively. How high is the tree? (Hint: Draw figure) Show the steps needed to find the height of the tree and give your answer in calculator ready form.

Question 6 [6 points] Establish the identity: \[
\frac{\sin^2 \theta + 4 \sin \theta + 3}{\cos^2 \theta} = \frac{3 + \sin \theta}{1 - \sin \theta}
\]
Question 7 [8 points] Graph the polar equations:

a) $r = 4 \sin(3\theta)$

b) $r = 3 + 2 \cos \theta$
Question 8 [10 points] Find the amplitude, period, and phase shift of $f(x) = -3\cos(2x - \frac{\pi}{2})$.
Then graph one period of the function showing the coordinates of 5 key points.