MAC 1114

EXAM FOUR

FALL-2016

Name

Signature

Problem	Total Points	Score
1	10	
2	15	
3	20	
4	15	
5	15	
6	10	
7	10	
8	15	
Total	110	

- You are not permitted to use a calculator on this exam.
- In order to receive full credit, you must show your work. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Good luck!

Note that $(a-b)^2 = a^2 - 2ab + b^2$

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1 (10 points) Verify the following identity.

We start from the left hand side

$$(\sin\theta - \cos\theta)^2 = 1 - \sin 2\theta$$

$$(\sin\theta - \cos\theta)^2 = \sin^2\theta - 2\sin\theta \cdot \cos\theta + \cos^2\theta$$

$$= \underbrace{\sin^2(\theta) + \cos^2(\theta)}_{=1 \text{ Pythagorean}} - 2\sin\theta \cdot \cos\theta$$

$$= 1 - 2\sin\theta \cdot \cos\theta$$

$$= 1 - \sin(2\theta)$$ because of the double angle formula of sine.

2 (15 points)

(a) (5 points) Find the exact value of $\sin(\sin^{-1}(-2))$

undefined

because if $\sin^{-1}(-2) = \theta$, then $\sin(\theta) = -2$ which is impossible.

(b) (5 points) Find the exact value of $\cos^{-1}(-\frac{\sqrt{3}}{2}) = \theta$

Let's call it θ.

Then $\cos(\theta) = -\frac{\sqrt{3}}{2}$ where $\theta \in [0, \pi]$.

So, θ has to be $\boxed{\frac{5\pi}{6}}$ because $\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$

Note that cosine is negative in the second quadrant.

(c) (5 points) Complete the following equality

$$\tan(70°) = \cot(\ ?\ )$$

$\tan(70°) = \cot(20°)$

by the cofunction idea.

Note that cosine is even and sine is odd function.

$\boxed{3}$ (20 points)

(a) (5 points) Find the rectangular coordinates of $(r, \theta) = (-4, -150°)$.

$X = r \cdot \cos\theta$

$x = -4 \cdot \cos(-150°)$

$X = -4 \cdot \cos(150°)$

$X = -4 \cdot (-\cos(30°))$

$X = 4 \cdot \cos(30°)$

$x = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$

$y = r\sin\theta$

$y = -4 \cdot \sin(-150°)$

$y = -4 \cdot (-\sin(150°))$

$y = 4 \cdot \sin(150°)$

$y = 4 \cdot \sin(30°)$

$y = 4 \cdot \frac{1}{2} = 2$

$(x, y) = (2\sqrt{3}, 2)$

(b) (5 points) Give another representation of $(r, \theta) = (-4, -150°)$.

Here are two representation:

$(4, -150° + 180°) = (4, 30°)$ and $(-4, -150° + 360°) = (-4, 210°)$

(c) (10 points)

Convert the following polar equation to a rectangular equation

$$r = 10 \sin\theta$$

Let's multiply both sides by r:

$r^2 = r \cdot 10\sin\theta$

$r^2 = 10 \cdot r\sin\theta$

$x^2 + y^2 = 10 \cdot y$ because $r^2 = x^2 + y^2$ and $y = r\sin\theta$

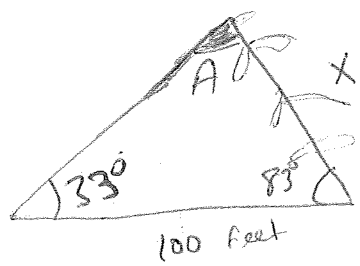
$x^2 + y^2 - 10y = 0$

$x^2 + y^2 - 10y + 25 = 25$ we added 25 to both sides

$x^2 + (y-5)^2 = 25$

which is a circle centered at $(0, 5)$ with radius 5.

4 (15 points) John wants to measure the height of a tree. He walks exactly 100 feet from the base of the tree and looks up. The angle from the ground to the top of the tree is 33°. This particular tree grows at an angle of 83° with respect to the ground rather than vertically (90°). How tall is the tree? (You can leave your answer in the calculator ready form.)



X is the heigth of the tree.
We can use Law of sine to find it

$A + 33° + 83° = 180°$

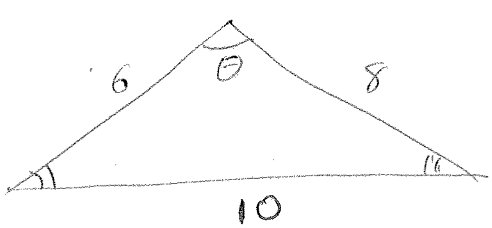
$A = 64°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{100}{\sin 64°} = \frac{X}{\sin 33°}$$

$$X = \frac{100 \cdot \sin 33°}{\sin 64°}$$ is the height of the tree.

5 (15 points) Find the largest angle of the triangle with $a = 6$, $b = 8$, and $c = 10$
You need to use either the law of sines or the law of cosines for this question, other methods will be disregarded. (You can leave your answer in the calculator ready form.)



θ has to be the largest angle in this triangle because it is the opposite of the largest side.

We use law of cosine:

$10^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cdot \cos(\theta)$

$100 = 36 + 64 - 96 \cdot \cos(\theta)$

$0 = -96 \cdot \cos(\theta)$

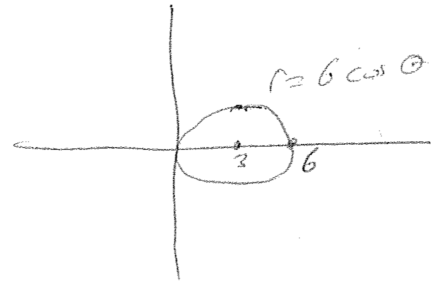
$\cos(\theta) = 0 \Rightarrow \theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$. But $\theta = \frac{3\pi}{2}$ is not possible because it is greater than 180°.

$\Rightarrow \boxed{\theta = \frac{\pi}{2}}$

6 (10 points) Identify the polar graph (line, circle, cardioid, limacon, rose): If a circle, name the center and the radius. If a limacon, name the type. If a rose, state the number of petals.

(a) (5 points) $r = 6\cos\theta$

it is a circle
with center $(3,0)$
and radius 3.



(b) (5 points) $r = 4\sin 3\theta$

it is a rose curve with 3 petals.

7 (10 points) Take the power of the following complex number. Write the answer in rectangular form.

$$[2(\cos 30° + i\sin 30°)]^5$$

By De Moivre's formula

$$= 2^5 \cdot [\cos(5 \cdot 30°) + i \cdot \sin(5 \cdot 30°)]$$

$$= 32 \cdot [\cos(150°) + i \cdot \sin(150°)]$$

$$= 32\,(-\cos(30°) + i \cdot \sin(30°))$$

$$= 32\left(-\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}\right)$$

$$= -16\sqrt{3} + i \cdot 16$$

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$\boxed{8}$ (15 points) Test for symmetry and graph the polar equation $r = 1 + \cos(\theta)$ (note that $\sqrt{3} \approx 1.7$ and $\sqrt{2} \approx 1.4$)

it is a cardioid

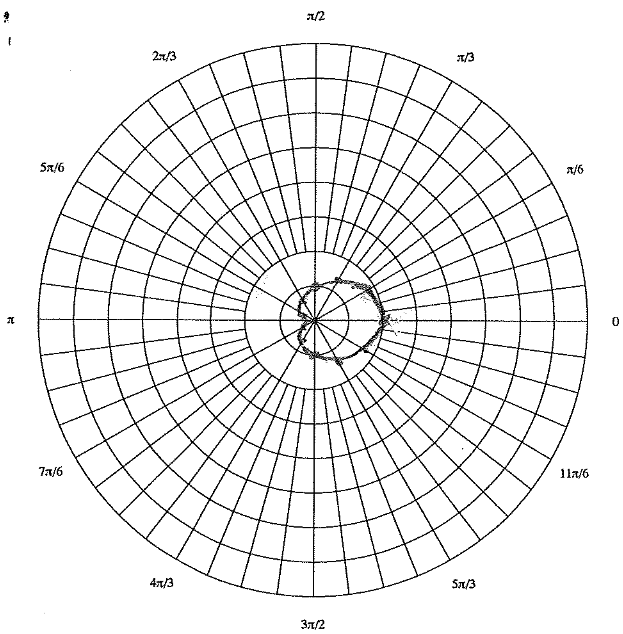
symmetry with respect to polar axis :

replace θ by $-\theta$

$r = 1 + \cos(-\theta)$

$r = 1 + \cos(\theta)$ ✓

it is symmetric about the polar axis.



symmetry w.r.t. the pole :

rerplace r by $-r$

$-r = 1 + \cos(\theta)$

$r = -1 - \cos(\theta)$ fails the test

symmetry w.r.t. the line $\theta = \frac{\pi}{2}$:

replace r by $-r$ and θ by $-\theta$

$-r = 1 + \cos(-\theta)$

$-r = 1 + \cos(\theta)$ fails the test.

$r = -1 - \cos(\theta)$

θ	r
0	$1+\cos(0) = 1+1 = 2$
$\frac{\pi}{6}$	$1+\cos(\frac{\pi}{6}) = 1 + \frac{\sqrt{3}}{2} \approx 1.85$
$\frac{\pi}{3}$	$1+\cos(\frac{\pi}{3}) = 1 + \frac{1}{2} = 1.5$
$\frac{\pi}{2}$	$1+\cos(\frac{\pi}{2}) = 1+0 = 1$
$\frac{2\pi}{3}$	$1+\cos(\frac{2\pi}{3}) = 1 - \frac{1}{2} = 0.5$
$5\pi/6$	$1+\cos(5\pi/6) = 1 - \sqrt{3}/2 \approx 0.15$
π	$1+\cos(\pi) = 1-1 = 0$