Part II

SHOW YOUR WORK (60 points: 8 questions). **All work must be shown to earn full credit. Organize your work and write neatly so it is clear what you do and why.**

Question 1 [5 points] Find $\sin(2\theta)$ if $\cos\theta = -\frac{4}{5}$ and $\pi \le \theta \le \frac{3\pi}{2}$

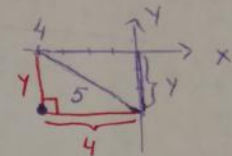
$$\sin(2\theta) = 2\underbrace{\sin\theta}_{?}\ \underbrace{\cos\theta}_{-\frac{4}{5}}$$

$\theta \in$ III quadrant, so $\sin\theta < 0$

$y = -\sqrt{5^2 - 4^2} = -\sqrt{9} = -3$

$\sin\theta = \frac{y}{r}$

$\sin\theta = -\frac{3}{5}$



$$\sin(2\theta) = 2\sin\theta\cos\theta = 2\cdot\left(-\frac{3}{5}\right)\cdot\left(-\frac{4}{5}\right) = \boxed{\frac{24}{25}}$$

Question 2 [10 points] If $\cot\theta = -\frac{1}{3}$, and $\sin\theta < 0$, find the exact values of each of the remaining trigonometric functions of θ. (you need to find $\tan\theta$, $\sin\theta$, $\cos\theta$, $\sec\theta$, $\csc\theta$).

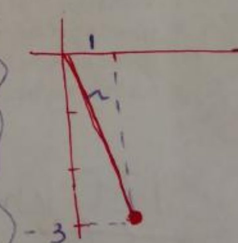
$\cot\theta = -\frac{1}{3}$ $\qquad \sin\theta < 0$

$\cot\theta = \frac{\cos\theta}{\sin\theta} < 0$, $\boxed{\sin\theta < 0}$ so $\boxed{\cos\theta > 0}$

$$\boxed{\tan\theta} = \frac{1}{\cot\theta} = \frac{1}{-\frac{1}{3}} = -\left(1 \div \frac{1}{3}\right) = -\left(1 \cdot \frac{3}{1}\right) = \boxed{-3}$$

$$\sin\theta = \frac{y}{r} = \boxed{\frac{-3}{\sqrt{10}}} = -\frac{3\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \boxed{\frac{1}{\sqrt{10}}} = \frac{\sqrt{10}}{10}$$



$r = \sqrt{(-3)^2 + 1^2} = \sqrt{10}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{\sqrt{10}}} = 1 \div \frac{1}{\sqrt{10}} = \boxed{\sqrt{10}}$$

7

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{3}{\sqrt{10}}} = 1 \div \left(-\frac{3}{\sqrt{10}}\right) = 1 \cdot \left(-\frac{\sqrt{10}}{3}\right) = \boxed{-\sqrt{}}$$

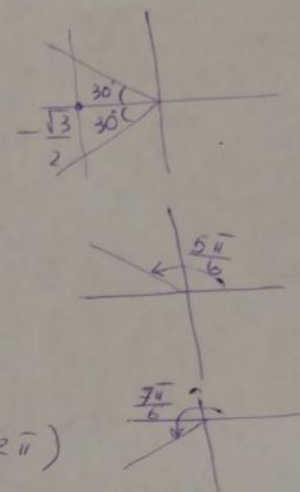
Form 1

Question 3 [8 points] Solve the equation $\cos(2x) = \underbrace{-\frac{\sqrt{3}}{2}}$ on the interval $0 \le x < 2\pi$.

reference angle is 30°

$2x = \frac{5\pi}{6} + 2\pi n$ or $2x = \frac{7\pi}{6} + 2\pi n$

$x = \frac{5\pi}{12} + \pi n$ or $x = \frac{7\pi}{12} + \pi n$



$x \in [0, 2\pi)$, so:

$\boxed{x = \frac{5\pi}{12}}$ $\boxed{x = \frac{7\pi}{12}}$

$x = \frac{5\pi}{12} + \pi = \boxed{\frac{17\pi}{12}}$ $x = \frac{7\pi}{12} + \pi = \boxed{\frac{19\pi}{12}}$

$x = \frac{5\pi}{12} + 2\pi \notin [0, 2\pi)$ $x = \frac{7\pi}{12} + 2\pi \notin [0, 2\pi)$

Question 4 [5 points] Write as an algebraic expression (without any trigonometric functions):

$\cos(\underbrace{\sin^{-1} x}_{\alpha} - \underbrace{\cos^{-1} y}_{\beta})$

$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta = \boxed{y\sqrt{1-x^2} + x\sqrt{1-y^2}}$

$\cos\alpha = \cos(\sin^{-1} x) = \sqrt{1 - (\underbrace{\sin(\sin^{-1} x)}_{x})^2} = \sqrt{1-x^2}$

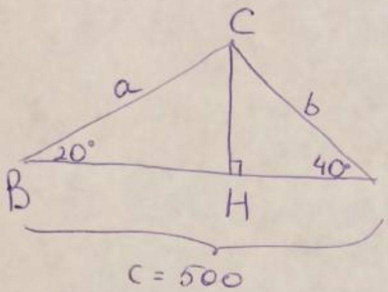
$\cos\beta = \cos(\cos^{-1} y) = y$

$\sin\alpha = \sin(\sin^{-1} x) = x$

$\sin\beta = \sin(\cos^{-1} y) = \sqrt{1 - (\underbrace{\cos(\cos^{-1} y)}_{y})^2} = \sqrt{1-y^2}$

Question 5 [8 points] Two people stand 500 feet apart. A tree that is perpendicular to the ground is on the line that separates these two people. The angles of elevation from each person to the top of the tree measure 20° and 40°, respectively. How high is the tree? (Hint: Draw figure)
Show the steps needed to find the height of the tree and give your answer in calculator ready form.

$CH = ?$



From $\triangle ABC$: $C = 180° - (20° + 40°) = 120°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$a = \frac{c \sin A}{\sin C} = \frac{500 \cdot \sin 40^\circ}{\frac{\sqrt{3}}{2}} = \frac{1000\sqrt{3}\sin 40^\circ}{3}$$

From $\triangle CBH$: $\sin 20^\circ = \frac{CH}{a}$

$$CH = a \sin 20^\circ = \boxed{\frac{1000\sqrt{3} \cdot \sin 40^\circ \cdot \sin 20^\circ}{3}}$$

Question 6 [6 points] Establish the identity: $\dfrac{\sin^2\theta + 4\sin\theta + 3}{\cos^2\theta} = \dfrac{3 + \sin\theta}{1 - \sin\theta}$

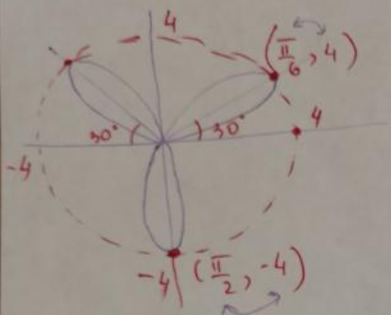
$$\underbrace{\frac{\sin^2\theta + 4\sin\theta + 3}{\cos^2\theta}}_{A} = \underbrace{\frac{3 + \sin\theta}{1 - \sin\theta}}_{B}$$

Work with B: $\frac{3 + \sin\theta}{1 - \sin\theta} \cdot \frac{1 + \sin\theta}{1 + \sin\theta} =$

$$= \frac{(3 + \sin\theta)(1 + \sin\theta)}{1^2 - \sin^2\theta} = \underbrace{\frac{3 + 4\sin\theta + \sin^2\theta}{\cos^2\theta}}_{\substack{\parallel \\ A}}$$

Question 7 [8 points] Graph the polar equations:

a) $r = 4\sin(3\theta)$ rose



points:

θ	r
$\frac{\pi}{12}$	$4\sin(3\cdot\frac{\pi}{12}) = 4\cdot\frac{\sqrt{2}}{2} = 2\sqrt{2}$
$\frac{\pi}{6}$	$4\sin(3\cdot\frac{\pi}{6}) = 4$
$\frac{\pi}{3}$	$4\sin(3\cdot\frac{\pi}{3}) = 0$
$\frac{\pi}{2}$	$4\sin(3\cdot\frac{\pi}{2}) = -4$
$5\pi/6$	$4\sin(3\cdot\frac{5\pi}{6}) = 4$

b) $r = 3 + 2\cos\theta$ limacon

$\frac{A}{B} = \frac{3}{2} > 1$ no inner loop

symmetry to polar axis:

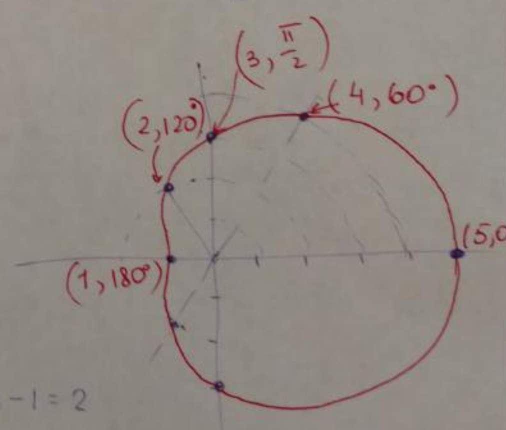
$r = 3 + 2\cos(-\theta) = \underbrace{3 + 2\cos(\theta)}$

the same as original equation

Points:

θ	r
$\frac{\pi}{6}$	$3 + 2\cos\frac{\pi}{6} = 3 + \cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}}$:
$\frac{\pi}{3}$	$3 + 2\cos\frac{\pi}{3} = 3 + 2\cdot\frac{1}{2} = 4$
$\frac{\pi}{2}$	$3 + 2\cos\frac{\pi}{2} = 3$
$\frac{2\pi}{3}$	$3 + 2\cos\frac{2\pi}{3} = 3 + 2\cdot(-\frac{1}{2}) = 3 - 1 = 2$
π	$3 + 2\cos\pi = 3 - 2 = 1$
0	$3 + 2\cdot\cos 0 = 3 + 2 = 5$

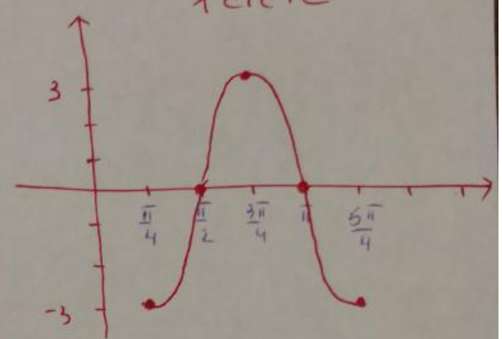
Then use symmetry



Question 8 [10 points] Find the amplitude, period, and phase shift of $f(x) = -3\cos(2x - \frac{\pi}{2})$. Then graph one period of the function showing the coordinates of 5 key points.

$f(x) = -3\cos(2x - \frac{\pi}{2})$

reflection across x-axis

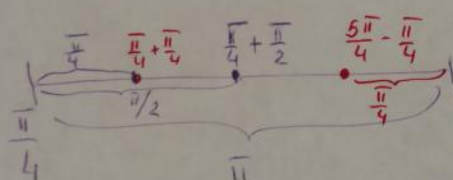


$A = 3$

Period $= \frac{2\pi}{2}$

Phase shift: $\frac{\pi}{2} \div 2 = \frac{\pi}{4}$

So, starting from $\frac{\pi}{4}$ graph 1 cicle that is π



$\frac{\pi}{4} + \pi = \frac{5\pi}{4}$

Points:

x	$f(x)$
$\pi/4$	$-3 \cdot \cos(2 \cdot \frac{\pi}{4} - \frac{\pi}{2}) = -3$
$\pi/4 + \frac{\pi}{4} = \frac{\pi}{2}$	0
$\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3\pi}{4}$	3
$\frac{\pi}{4} + \frac{3\pi}{4} = \pi$	0
$\frac{\pi}{4} + \frac{5\pi}{4} = \frac{5\pi}{4}$	-3

solution to Multiple Choice

1 C
2 D
3 A
4 E
5 D
6 D
7 B
8 D
9 A
10 B
11 C
12 E
13 D
14 C
15 A
16 A
17 B
18 D
19 C
20 D