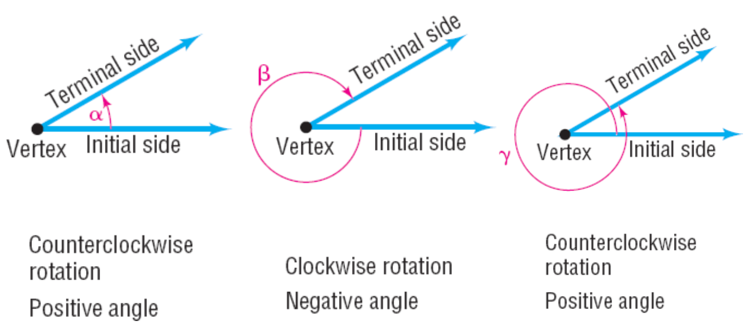
5.1 Angles and Their Measure

Definition

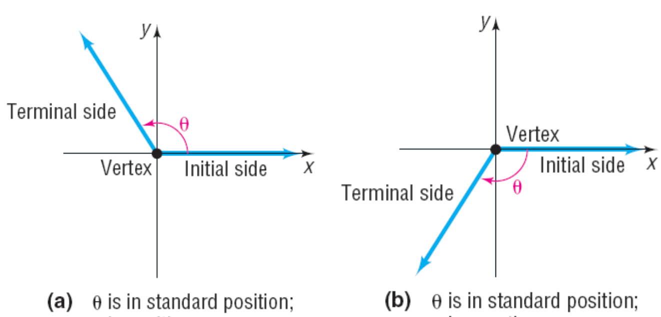
A **ray**, or **half-line**, is that portion of a line that starts at a point V on the line and extends indefinitely in one direction. The starting point V of a ray is called its **vertex.**

If two rays are drawn with a common vertex, they form an **angle.** We call one ray of an angle the **initial side** and the other the **terminal side.** If the rotation is in the counterclockwise direction, the angle is **positive;** if the rotation is clockwise, the angle is **negative.**



Definition

An angle ϑ is said to be in **standard position** if its vertex is at the origin of a rectangular coordinate system and its initial side coincides with the positive x-axis.



(a) θ is in standard position; θ is positive **(b)** θ is in standard position; θ is negative

When an angle ϑ is in standard position, the terminal side will lie either in a quadrant, in which case we say that ϑ **lies in that quadrant**, or the terminal side will lie on the x-axis or the y-axis, in which case we say that ϑ is a **quadrantal angle.**

Definition

The two commonly used measures for angles are *degrees* and *radians* from initial to terminal.

The angle formed by rotating the initial side exactly once in the counterclockwise direction until it coincides with itself (1 revolution) is said to measure 360 degrees. A **right angle** is an angle that measures 90°, or 1/4 revolution; a **straight angle** is an angle that measures 180°, or 1/2 revolution.

1. Draw each angles:

   (a) 45° (b) − 90 ° (c) 225° (d) 405°

Definition

A **central angle** is a positive angle whose vertex is at the center of a circle. The rays of a central angle subtend (intersect) an arc on the circle. If the radius of the circle is *r* and the length of the arc subtended by the central angle is also *r*, then the measure of the angle is **1 radian.**

For a circle of radius 1, the rays of a central angle with measure 1 radian subtend an arc of length 1. For a circle of radius 3, the rays of a central angle with measure 1 radian subtend an arc of length 3.

Theorem

For a circle of radius *r*, a central angle of ϑ radians subtends an arc whose length *s* is
$s = r\theta$

2. Find the length of the arc of a circle of radius 4 meters subtended by a central angle of 0.5 radian.

Converting Degrees to Radians and Radians to Degrees

Because the circumference of a circle of radius *r* equals $2\pi r$, we substitute $2\pi r$ for *s* in equation. We get $2\pi r = r\theta$, which implies $\theta = 2\pi$.

1 revolution= 360 ° = 2 π radians

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

3. Convert each angle in degrees to radians:
   (a) 30° (b) 120° (c) – 60°
4. Convert each angle in radians to degrees:
   (a) $\frac{\pi}{3}$ radian (b) $-\frac{\pi}{2}$ radian (c) $\frac{5\pi}{6}$ radians (d) 5 radians

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		210°	225°	240°	270°	300°	315°	330°	360°
Radians		$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

Definition

Two angles in standard position are said to be **coterminal** if they have the same terminal side.

5. Find the exact value of each of the following:

(a) sin 390°
(b) cos 420°
(c) tan 9 π/4
(d) sec (– 7 π/4)
(e) csc (– 270 °)