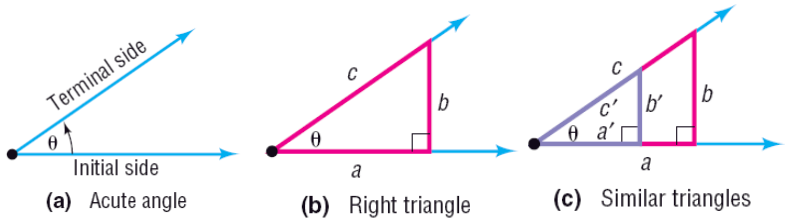
5.2 Right Triangle Trigonometry

Definition

A triangle in which one angle is a right angle (90°) is called a **right triangle.** Recall that the side opposite the right angle is called the **hypotenuse**, and the remaining two sides are called the **legs** of the triangle.

Pythagorean Theorem tells us that $c^2=a^2+b^2$.

Applying to Standard Position Angles



Using the three sides of this triangle, we can form exactly six ratios:

$$b/c, a/c, b/a, c/b, c/a, a/b$$

Any two right triangles formed using the angle θ will be similar and, hence, corresponding ratios will be equal. As a result,

$$b/c=b'/c' \quad a/c=a'/c' \quad b/a=b'/a' \quad c/b=c'/b' \quad c/a=c'/a' \quad a/b=a'/b'$$

Definition

The six ratios of the lengths of the sides of a right triangle are called **trigonometric functions of acute angles** and are defined as follows:

Function Name	Abbreviation	Value	Function Name	Abbreviation	Value
sine of θ	$\sin\theta$	$\frac{b}{c}$	cosecant of θ	$\csc\theta$	$\frac{c}{b}$
cosine of θ	$\cos\theta$	$\frac{a}{c}$	secant of θ	$\sec\theta$	$\frac{c}{a}$
tangent of θ	$\tan\theta$	$\frac{b}{a}$	cotangent of θ	$\cot\theta$	$\frac{a}{b}$

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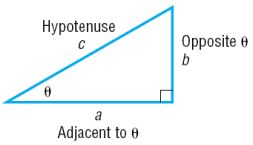
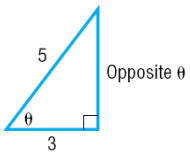
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{c} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{c} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{c}{b} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{c}{a} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{a}{b}$$



Examples:

1. Find the value of each of the six trigonometric functions of the angle ϑ



2. Find the exact values of the six trigonometric functions of $\pi/4 = 45°$
3. Find the exact values of the six trigonometric functions of $\pi/6 = 30°$ and $\pi/3 = 60°$

θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Reciprocal & Quotient Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

4. Given $\sin\vartheta=8/17$, $\cos\vartheta=15/17$ find the value of each of the four remaining trigonometric functions of ϑ using identities.

Pythagorean Identities

$$\sin^2\vartheta+\cos^2\vartheta=1$$
$$\tan^2\vartheta+1=\sec^2\vartheta$$
$$\cot^2\vartheta+1=\csc^2\vartheta$$

5. Find the exact value of each expression.

   a. $\tan 50°-(\sin 50°/\cos 50°)$
   b. $\sin(2\pi/12)+1/(\sec(2\pi/12))$

Finding Values of Trig Functions when One is Unknown

Given the value of one trigonometric function of an acute angle ϑ, the exact value of each of the remaining five trigonometric functions of ϑ can be found in either of two ways.

Method 1 Using the Definition

- **STEP 1:** Draw a right triangle showing the acute angle ϑ.
- **STEP 2:** Two of the sides can then be assigned values based on the value of the given trigonometric function.
- **STEP 3:** Find the length of the third side by using the Pythagorean Theorem.
- **STEP 4:** Use the definitions in equation (1) to find the value of each of the remaining trigonometric functions.

Method 2 Using Identities

Use appropriately selected identities to find the value of each of the remaining trigonometric functions.

6. Given $\tan\vartheta=1/2$, ϑ an acute angle, find the exact value of each of the remaining five trigonometric functions of ϑ.

Definition

Two acute angles are called **complementary** if their sum is a right angle, or 90°.

The functions sine and cosine, tangent and cotangent, and secant and cosecant are called **cofunctions** of each other.

Complementary Angle Theorem

Cofunctions of complementary angles are equal.

Complementary angles
$\sin 30^\circ = \cos 60^\circ$
Cofunctions

Complementary angles
$\tan 40^\circ = \cot 50^\circ$
Cofunctions

Complementary angles
$\sec 80^\circ = \csc 10^\circ$
Cofunctions

θ (Degrees)	θ (Radians)
$\sin\theta = \cos(90^\circ - \theta)$	$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$
$\cos\theta = \sin(90^\circ - \theta)$	$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$
$\tan\theta = \cot(90^\circ - \theta)$	$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$
$\csc\theta = \sec(90^\circ - \theta)$	$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$
$\sec\theta = \csc(90^\circ - \theta)$	$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$
$\cot\theta = \tan(90^\circ - \theta)$	$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

7. Find the exact value of each expression. Do not use a calculator.

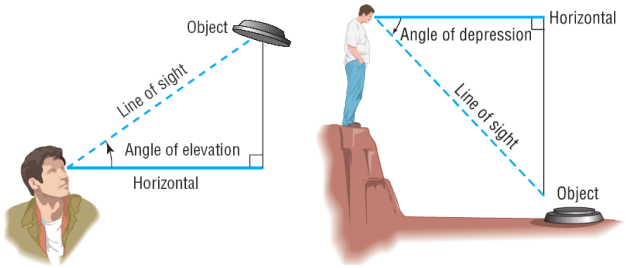
(a) sec 28°–csc 62°

(b) sin 35°/cos 55°

Definition

Vertical heights can sometimes be measured using either the *angle of elevation* or the *angle of depression*. If a person is looking up at an object, the acute angle measured from the horizontal to a line of sight to the object is called the **angle of elevation.**

If a person is standing on a cliff looking down at an object, the acute angle made by the line of sight to the object and the horizontal is called the **angle of depression.**



8. The tallest tower built before the era of television masts, the Eiffel Tower was completed on March 31, 1889. Find the height of the Eiffel Tower (before a television mast was added to the top) using the information given in the illustration. [Angle measuring 85.361 deg and ½ base measuring 80ft]
9. To measure the height of Lincoln's caricature on Mt. Rushmore, two sightings 800 feet from the base of the mountain are taken. If the angle of elevation to the bottom of Lincoln's face is 32° and the angle of elevation to the top is 35°, what is the height of Lincoln's face?